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Graph of an affine function: a study from the perspective of the Potential Didactic Contract

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ARTICLE

Abstract: The research aimed to investigate the potential didactic contract through expectations, potential contract rules, and indications of rupture present in a 1st-grade High School Mathematics textbook regarding the knowledge of the graph of the affine function. The technique adopted was document analysis, taking as a reference the definition of the elements that compose this contract. A total of three expectations, one potential explicit rule, three potential implicit rules, and two indications of rupture were identified. The results point to regularities that show a tendency towards standardization of the procedures for constructing the graph of the affine function, revealing both the strength of these regularities and their possible limits for the production of broader mathematical meanings.

Keywords: Textbook. Potential Didactic Contract. Graph of Affine Function.

Gráfico de funciones afines: un estudio desde la perspectiva del Contrato Didáctico Potencial

Resumen: La investigación tuvo como objetivo investigar el contrato didáctico potencial a través de expectativas, reglas de contrato potencial e indicios de ruptura presentes en un libro de texto de Matemáticas de 1er año de Secundaria respecto del conocimiento del gráfico de la función afín. La técnica empleada fue el análisis documental, tomando como referencia la definición de los elementos que componen dicho contrato. Se identificaron tres expectativas, una posible regla explícita, tres posibles reglas implícitas y dos indicios de ruptura. Los resultados señalan regularidades que muestran una tendencia hacia la estandarización de los procedimientos para la construcción de la gráfica de la función afín, revelando tanto la fuerza de estas regularidades como sus posibles límites para la generación de significados matemáticos más amplios.

Palabras clave: Libro de Texto. Potencial Contrato Docente. Gráfico de Funciones Afines.

Gráfico da função afim: um estudo sob a perspectiva do Contrato Didático Potencial

Resumo: A pesquisa teve como objetivo investigar o contrato didático potencial por meio das expectativas, regras de contrato potenciais e indícios de ruptura presentes em um livro didático de Matemática do 1º ano do Ensino Médio no tocante ao saber gráfico da função afim. A técnica de análise adotada foi a análise documental, fundamentada na definição dos elementos que estruturam tal contrato. Foi identificado um total de três expectativas, uma regra explícita potencial, três regras implícitas potenciais e dois indícios de ruptura. Os resultados apontam para regularidades que evidenciam uma tendência à padronização dos procedimentos de construção do gráfico da função afim, revelando tanto a força dessas regularidades quanto seus possíveis limites para a produção de significados matemáticos mais amplos.

Palavras-chave: Livro Didático. Contrato Didático Potencial. Gráfico da Função Afim.

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1 Introduction

According to Turíbio and Silva (2017), despite technological advances and the wide variety of information sources currently available, textbooks are still an essential teaching resource in Brazilian classrooms. This is because they remain the main teaching material used in the classroom, accessible to all students simultaneously, which makes them a widely used tool.

For Carvalho and Lima (2010), textbooks occupy a prominent position in mathematics classes, maintaining close relationships with both teachers, by assisting in the planning of their classes, and with students, by serving as a support for their learning. In addition, textbooks are seen as representatives of knowledge, carrying with them official aspects regarding their teaching.

Eloi and Andrade (2020) argue that, considering the fact that textbooks establish close relationships with the teacher, the student, and knowledge, their approach can influence how this knowledge is worked on by the teacher in the classroom. This idea is supported by studies such as those by Barbosa (2017) and Santos (2015), which comparatively analyzed this resource and the approach of the teacher, as a user of the book, to a given subject. The results of the aforementioned research indicate that, to a certain extent, there are similarities between both approaches.

Given the relevance of textbooks and their potential influence on the approach to knowledge in the classroom, it becomes necessary to investigate them from the perspective of different theoretical frameworks. In this context, studies on textbooks in light of the notion of a potential didactic contract stand out. This is a theoretical proposition presented in Eloi and Andrade (2020), based on the ideas of Brousseau (1986), and refers to a hypothetical didactic contract present in the perspective assumed by the textbook.

The didactic contract was originally defined by Brousseau (1986) as a theoretical element of the Theory of Didactic Situations. This term comprises the set of expectations and rules that condition the functioning of the didactic relationship, that is, the relationship established between the teacher, the student, and knowledge. Although the didactic contract is defined in terms of the relationships established within the classroom, it is considered that the textbook approach brings with it a potential didactic contract, intended for hypothetical teachers and students who are users of this didactic resource (Eloi and Andrade, 2020).

The proposition of the potential didactic contract is based on the fact that the textbook carries choices regarding certain knowledge (Carvalho and Lima, 2010). These choices are structured considering hypothetical teachers and students — users of the textbook — and present a specific organization of how knowledge will be addressed in the classroom. Thus, it is believed that this teaching resource contains *seeds* of how knowledge can be negotiated in the teaching relationship, with regularities, *breaks* in regularities, and elements that can facilitate the tasks assigned to students.

In this context, Eloi and Andrade (2020) argue that there is a connection between the elements of the textbook and the contractual elements discussed in the literature. They also define the elements of a potential didactic contract as expectations, potential contract rules, indications of breaches, and indications of the effects of the contract.

The concept of function arose from the need to explain, understand, and predict natural phenomena. Eves (2011) highlights that the importance of the ideas underlying function goes beyond mathematics, also covering other areas of knowledge in which quantity and measurements play a fundamental role, such as in the natural sciences. Therefore, function is considered to be extremely relevant knowledge for the Basic Education curriculum, as it is closely linked to the explanation of everyday phenomena involving relationships of dependence between variables.

According to current curriculum guidelines, the concept of function permeates the entire Basic Education pathway, from the early years of Elementary School, with the development of intuitive notions of function, to High School, when these notions are deepened and formalized (Brasil, 2018; Pernambuco, 2019, 2021).

However, when dealing specifically with the affine function, several studies (Santos, 2002; Dornelas, 2007; Delgado, 2010; Santana, 2016; Tozo and Oliveira, 2016) point out recurring difficulties in the learning process of this knowledge at different levels of education.

Tozo and Oliveira (2016), in particular, highlight the difficulty students have in constructing the graph of the linear function. These difficulties become even more evident when analyzing student performance in large-scale assessments, such as the Pernambuco Basic Education Assessment System (SAEPE).

By systematically examining the results of the SAEPE between 2008 and 2018, Santana (2022) observes that, in all editions of this assessment, students performed poorly in skills related specifically to the linear function, such as recognizing and analyzing the graph of this function.

In view of the discussion presented so far, it is considered extremely relevant to develop research that investigates the different nuances of teaching and learning the linear function, especially the concepts underlying its graph. In this sense, we reaffirm the recognition of the textbook as a central element in mathematics classes, since it can directly influence the ways in which knowledge is taught and learned. Thus, this study proposes to investigate the approach to the graph of the linear function in a 10th-grade textbook.

To this end, the notion of potential didactic contract is adopted as a theoretical reference, from which we seek to answer the following question: how is the potential didactic contract established in the approach to the graph of the linear function in a 1st-garde High School textbook? In this context, the main objective of this research is to analyze the potential didactic contract through expectations, potential contract rules, and evidence of ruptures present in a 1st-year high school textbook regarding the knowledge of linear function graphs[1].

The following sections comprise the theoretical foundation in which a discussion about the potential didactic contract is woven; the methodology in which the procedures adopted to carry out this study are described; the analysis of the potential didactic contract; and, finally, the final considerations.

2 Theoretical basis

As already indicated in the previous section, the notion of potential didactic contract refers to a possible didactic contract present in the textbook's approach. In this sense, for the reader's better understanding, it is deemed necessary to address, at first, the ideas that underpinned the notion of didactic contract proposed by Brousseau (1986) and, subsequently, the propositions presented in Eloi and Andrade (2020) regarding the potential didactic contract.

In general terms, the didactic contract refers to the explicit and implicit rules that govern the functioning of the didactic relationship, that is, the relationship established between teachers and students in the process of knowledge acquisition. According to Chevallard, Bosch and Gascón (2001), a didactic relationship is established when some people encounter a problem

[1] This article is the result of a master's research project that investigated the relationships between the didactic contract and the potential didactic contract when the related function of knowledge is involved (Eloi, 2019).

whose answer is not obvious and decide to act to solve it. In this context, people assume the role of students and may seek the help of a coordinator, in this case, the teacher. This relationship is represented in the literature by a geometric figure, the Triangle of Didactic Situations (Brousseau, 1986), as shown in Figure 1.

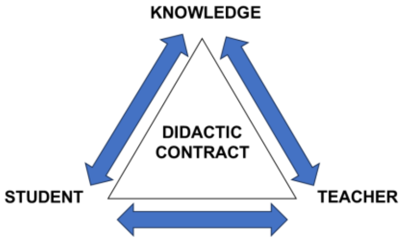


Figure1: Triangle of Didactic Situations (Eloi and Andrade, 2020, p. 236)

Thus, the didactic contract can be understood as the teacher's expectations regarding the behavior of their students, as well as the students' expectations regarding their teacher's actions in the management of knowledge. These actions and behaviors are, for the most part, defined implicitly, determining the rights and duties of the partners in the didactic relationship throughout the process of knowledge acquisition.

In the words of Brousseau (1986), the teaching contract is defined as

> set of teacher behaviors that are expected by students and the set of student behaviors that are expected by the teacher. This contract is the set of rules that determine, explicitly to a small extent, but mainly implicitly, what each partner in the teaching relationship should manage and what, in one way or another, they will have to account for to the other. This system of reciprocal obligations resembles a contract. What interests us is the teaching contract, that is, the part of the contract that is specific to the content: the mathematical knowledge targeted (Brousseau, 1986, p. 51).

Sarrazy (1995) argues that the didactic contract is based on the habits that the teacher reproduces in their teaching practice, consciously or unconsciously. The repetition of these habits creates a regularity that, over time, generates mutual expectations between teacher and student. In the same vein, Bessot (2003) points out that the didactic contract involves a set of specific requirements related to each activity, reinforcing the idea that there are rules that guide the actions of teachers and students.

Expanding on this understanding, in Eloi and Andrade (2020) we propose that contractual rules can be understood as patterns and regularities that emerge over time, originating from the habits established in the interactions between teachers and students.

The rules that make up the didactic contract can be classified as explicit or implicit. Explicit rules are those communicated clearly and directly by the parties involved, usually when knowledge is at stake (Jonnaert and Borght, 2002). An example of this is when, in an algebra class, the teacher states that the unknown must always be represented by the letter x. By establishing this convention clearly and directly, he institutes a regularity that can be understood as an explicit rule.

On the other hand, implicit rules are not explicitly verbalized, but are constructed subliminally throughout the teacher's practice. According to Almeida (2016), these rules emerge from the regularities of the teacher's approach to a particular piece of knowledge, becoming so recurrent that they are intuitively recognized in the didactic relationship. For example, if, when teaching the content of quadrilaterals, the teacher always presents activities that are accompanied by illustrative images of the quadrilaterals covered, students come to expect that every activity related to this knowledge will present an image, even if this has not been explicitly stated. This situation, from the perspective of the didactic contract, can be understood as an implicit rule.

As previously stated, the didactic contract is mostly implicit. Therefore, its elements become evident, especially in moments of contractual rupture. In other words, when one of the partners in the didactic relationship (teacher and students) acts contrary to the established rules.

For Almeida (2016), these moments of breach elucidate essential aspects of the contract: the student is surprised by not knowing how to resolve a given situation and may show frustration at not having been led to success by the teacher. In turn, the teacher is disappointed with the student's failure, believing that they have provided the necessary means for their learning.

Almeida and Lima (2017) state that the teaching contract cannot be fully explained or negotiated in advance between teacher and students, as this would compromise its functioning. In other words, the clauses and attitudes that lead to ruptures cannot be predicted or determined in advance. Brousseau (1986) complements this thought, stating that ruptures are overcome with knowledge and are therefore inevitable and unpredictable.

To illustrate the concept of contractual breach, we return to the example of the implicit rule discussed earlier regarding *knowledge of quadrilaterals*. The breach occurs when the teacher proposes an activity on this content, but without the corresponding image of the quadrilateral in question. In this case, contrary to their expectations, students may express surprise or difficulty. From that moment on, it becomes clear that there was, in fact, a collective expectation that activities related *to quadrilaterals* would always be accompanied by an image, thus revealing the existence of an implicit rule.

Once the fundamental ideas regarding the didactic contract have been addressed, the following paragraphs discuss the potential didactic contract.

The notion of the potential didactic contract arises from reflections on the ideas proposed by Brousseau (1986) and on the structure of the textbook. As discussed in Eloi and Andrade (2020), the textbook brings with it choices that concern how knowledge should be approached in the classroom, that is, it contains *seeds* of what can be negotiated between teacher and students.

> The potential didactic contract comprises a didactic contract present in the textbook's approach. The adoption of the word potential in the composition of the term is justified by the fact that this contract is defined in terms of negotiation intentions, with the expectation that teachers and students who use the textbook will "reproduce" this contract in the classroom. Therefore, this term comprises a contract with the potential to be established in the classroom (Eloi and Andrade, 2020, p. 239).

Although the idea of a relationship between the didactic contract and the textbook is an original contribution of the study presented in Eloi and Andrade (2020), it already appears in the works of Bessot and Hoai An (1994) and Santos (2005). While these studies do not directly define the potential didactic contract, some elements of this notion already permeate their analyses.

Both studies analyzed how students solve problems that break the rules in the textbook. The results revealed two types of behavior: either students try to force the application of the rules in the book, or they encounter difficulties in solving the activities. Such behaviors indicate a relationship between the rules of the didactic contract in the textbook and those under which students actually operate.

The above-mentioned works support the hypothesis that there is a division of responsibilities in the textbook: the lesson text would be the responsibility of the teacher, while the exercises would be assigned to the students. The interactions between these parts reveal, to a certain extent, a didactic contract, since they establish actions legitimately required by the teacher and recognized by the students who use the material.

According to Brousseau (1986), in the context of the didactic contract, the teacher is expected to provide sufficient conditions for the student to acquire knowledge, and the student, in turn, is expected to meet the conditions initially proposed by the teacher.

Based on this understanding, in Eloi and Andrade (2020), we articulate the works of Bessot and Hoai An (1994), Brousseau (1986), and Santos (2005) and argue that, from the perspective of the potential didactic contract, the lesson text, under the responsibility of a hypothetical teacher, should provide the conditions for the student's appropriation of knowledge. Analogously, the exercises, the responsibility of the hypothetical student, should be solved based on the conditions established in the lesson text.

As a result of discussions around the potential didactic contract, in Eloi and Andrade (2020) propose the elements of the potential didactic contract, understood as a reinterpretation of the elements of the didactic contract already discussed in the literature, in light of the structure of the textbook. These elements are expectations, explicit and implicit potential contract rules, signs of breaches, and signs of contract effects[2] . As already indicated in the introduction, this work focuses its analysis on expectations, potential contract rules, and signs of breaches. Each of these elements is discussed below.

Expectations. Textbooks are designed with teachers and hypothetical students in mind, and in this context, they carry expectations, that is, intentions for negotiation about how knowledge should be approached in the classroom (Eloi e Andrade, 2020). According to the authors, these expectations are contained in the teacher's manual, since this material provides guidelines for the teacher's practice based on the use of the textbook, thus revealing the intentions of how this work should be conducted.

Potential contract rules. As already mentioned, contract rules are originally defined as the standards and regularities established in the teacher's practice, based on the establishment of habits. Potential contract rules can be understood as regularities in the approach to certain knowledge in the textbook, that is, they are so recurrent that they will be part of the hypothetical teacher's habit and will constitute part of the hypothetical student's set of expectations regarding that teacher's behavior. As in the definition of contract rules already existing in the literature, potential contract rules are divided into implicit and explicit in nature (Eloi and Andrade, 2020).

Potential explicit rules. These are clearly stated in the lesson text, generalizing procedures and establishing regularities in relation to certain knowledge. As an example, Eloi and Andrade (2020) discuss an excerpt from the first volume of the collection *Matemática: Contextos & Aplicação [Mathematics: Contexts and Applications]* (Dante, 2014), which states: “Problems involving proportionality can generally be solved using a linear function, which is why we state that the linear function is the mathematical model for proportionality problems” (Dante, 2014, p. 91).

The authors consider that this statement makes explicit a regularity: “most proportionality problems can be solved using a linear function” (Eloi and Andrade, 2020, p.

[2] We chose not to address the evidence of contract effects in this work.

243). This characterizes a potential explicit rule.

Potential explicit rules. These are rules clearly stated in the textbook, generalizing procedures and establishing regularities in relation to a given area of knowledge. As an example, in Eloi and Andrade (2020) we discuss an excerpt from the first volume of the collection *Matemática: Contextos & Aplicação* (Dante, 2014), which states that "problems involving proportionality, in general, can be solved using a linear function, and therefore we affirm that the linear function is the mathematical model for proportionality problems" (Dante, 2014, p. 91). This statement explicitly establishes a regularity: "most proportionality problems can be solved using a linear function" (Eloi and Andrade, 2020, p. 243). This characterizes a potential explicit rule.

Potential implicit rules. In Eloi and Andrade (2020, p. 244), potential implicit rules are defined as "regularities, present in the approach to knowledge, that are not explicitly mentioned". According to the authors, these rules are identified from the regularities imposed in the lesson text, constituting part of the habits of the hypothetical teacher.

Even so, according to the authors, it is possible to identify in the textbook certain breaks from previously established regularities or moments when these regularities are no longer followed. Potential contract rules derive from regularities in the approach to knowledge. Thus, the breaking of these regularities can be interpreted as an indication of a breach.

To illustrate this phenomenon, Eloi and Andrade (2020) revisit the previously discussed potential implicit rule found in the second volume of the collection *Matemática: Contextos & Aplicação* (Dante, 2014). According to their analysis, although the lesson text establishes the potential implicit rule, which defines the use of the letters x, y, z, and w as representations of unknowns in linear systems, there are exercises in which this convention is not maintained. Such situations constitute signs of rupture, as they break with a previously established regularity.

The following section presents the methodology adopted in this study.

3 Methodology

As mentioned in the opening lines, this article is an excerpt from a broader study (Eloi, 2019), whose objective was to analyze the relationships between the didactic contract established between teacher and students in a 1st-year high school class in the approach to linear function knowledge and the potential didactic contract established in the textbook adopted by the teacher.

In this context, this study corresponds to the analysis of the textbook used in the investigated reality, which is the first volume of the collection *Matemática: Contextos & Aplicação*, by Luiz Roberto Dante, 3rd edition, published in 2016.

The study of linear functions is addressed in chapter 3, entitled *Linear Functions and Modular Functions*. The analysis considered the section related to the graph of the affine function, entitled *Graph of the Affine Function f(x) = ax + b*, as well as an excerpt from the Teacher's Manual: *Observations and suggestions for the units and chapters*[3].

Documentary analysis was used as a data analysis technique, which, according to Sá-Silva, Almeida, and Guindane (2009), is a procedure that uses methods and techniques to analyze documents of various types through processes of apprehension and comprehension. Thus, it is understood that this technique is aligned with the objective of this study, since the analysis of the potential didactic contract presupposes the researcher's immersion in the

[3] When referring to this topic, only the *Teacher's Manual* will be mentioned.

universe studied, that is, in the textbook, in order to extract as much relevant information as possible.

As mentioned earlier, the analysis covers topics in the textbook related to the graph of the linear function and the teacher's manual. In these topics, the guidelines in the manual and all elements of the approach to knowledge (terms, definitions, examples, and exercises) were examined to identify elements of the potential didactic contract. The analysis was divided into three main areas: analysis of the teacher's manual, analysis of the lesson text, and analysis of the exercises. The focus of each of these areas is described in Table 1.

Table 1: Areas for analysis of the Potential Didactic Contract

Axis of Analysis	**Description**
Analysis of the Teacher's Manual	This corresponds to the analysis of the topics in the teacher's manual that contain teaching guidelines for working with each of the chapters and solving the exercises. That said, this area sought to identify expectations regarding the teacher's practice guidelines and expectations regarding the students' development of the exercises.
Analysis of the Lesson Text	According to the perspective adopted in this study, the lesson text would be the responsibility of a hypothetical teacher. In this sense, this axis sought to identify potential explicit and implicit rules and signs of disruptions.
Analysis of the Exercises	The exercises are hypothetically the responsibility of the student. Therefore, in this axis, we pointed out whether the exercises strengthened or broke the regularities established in the lesson text and whether their elements favor the establishment of signs of breaks.

Source: Own elaboration

The establishment of the potential teaching contract was analyzed through the interrelation between the three axes mentioned above. Thus, the implications of the expectations contained in the teacher's manual in the approach to knowledge in the lesson text, as well as in the exercises, were analyzed. The interrelationships between the lesson text and the exercises were also analyzed to observe whether the regularities established in the lesson text were followed in the exercises.

Each element of the potential didactic contract was assigned a code consisting of three capital letters referring to the name of each one. To organize elements of the same nature, a natural number was added to the code to indicate the order in which the element was identified in the analysis. Thus, we have: EXP.n (nth expectation identified), REP.n (nth potential explicit rule identified), RIP.n (nth potential implicit rule identified), and IRP.n (nth indication of rupture identified). Table 2 presents the criteria adopted for the analysis of the textbook.

Table 2: Criteria for analyzing the textbook

y Potential Didactic Contract Element	**Description**
Expectations	These are the guidelines contained in the teacher's manual regarding teaching practice.
Potential explicit rules	These rules arise from explicit references that generalize procedures and define regularities in the textbook regarding the approach to knowledge. The aim is to identify these references in the excerpt from the textbook referring to the

	lesson text.
Potential implicit rules	These rules are established based on regularities present in the approach to knowledge that are not explicitly stated, but are part of the hypothetical teacher's habit. These rules are identified based on the regularities present in the lesson text and, subsequently, the exercises are checked to see if these rules were reinforced or broken.
Evidence of breaches	*Breaches* in the regularities imposed in the approach to knowledge established in the textbook. These signs can be identified in the exercises when the regularities established in the lesson text are not observed, as well as in the lesson text itself, when, after the establishment of regularities, that is, the appearance of a certain number of regularities, elements that do not comply with them are presented.

Source: Own elaboration

The following section presents an analysis of the potential didactic contract.

4 Analysis of the potential didactic contract

The analysis of the potential didactic contract was divided into three structural axes: teacher's manual, lesson text, and exercises. The results from these axes are presented below.

4.1 Teacher's Manual

The analysis of the teacher's manual sought to identify expectations regarding working with the graph of the affine function. The textbook reflects intentions to negotiate how knowledge should be approached in the classroom. These intentions, in the context of the potential didactic contract, are the expectations found in the teacher's manual. Three expectations were identified: EXP.1, EXP.2, and EXP.3.

Expectation EXP.1 refers to the manual's intention to explore the implications of the rate of change and the initial value in the linear function graph. The manual highlights that working with the linear function graph is a relevant moment for the institutionalization of the rate of change (angular coefficient, a) and the initial value (linear coefficient, b).

According to the manual, the analysis of graphs of functions with variation of the coefficient a should be proposed so that students understand the relationship between this coefficient and the rate of change: “Students could analyze some graphs to understand the relationship between the rate of change and the coefficient a, such as the graphs of the functions$f(x) = x$,$g(x) = 3x$,$h(x) = 0{,}5x, i(x) = -x$,$j(x) = -2x$” (Dante, 2016, p. 333). By proposing this analysis and highlighting the relationship between the rate of change and the coefficient a, the expectation is that students will understand that the value of a determines the direction of growth of the function, that is, the rate of change, and, consequently, the slope of the line in the graph.

The manual also seeks to ensure that students understand that the coefficient b, that is, the initial value, determines the point of intersection of the graph with the Oy axis. This concern is evident in the following excerpt: “It would also be interesting for students to analyze some graphs, such as$f(x) = x$,$g(x) = x + 2$,$h(x) = x + 3$,$i(x) = x - 2$,$j(x) = x - 4$” (Dante, 2016, p. 333). These functions have the same rate of change but different initial values, which allows students to realize that the value of b determines the ordinate of the point of intersection with the Oy axis.

Given the above, it is understood that working with the graph of the linear function in the textbook is permeated by the expectation that the influences of the rate of change and initial value on the construction of the graph will be explored. The rate of change is related to the slope of the line, and the initial value determines the point of intersection with the *Oy* axis (EXP.1).

The teacher's manual also highlights the importance of working on understanding the terms *ascending line*, *descending line*, *identity function,* and *constant function*. The growth and decay of an affine function are solely related to the coefficient *a*. The manual suggests using mathematical reasoning to formally represent the relationships of growth and decay. For example: "$f(x) = 2x + 1$ is increasing, because if$a > 0$, the function is increasing" (Dante, 2016, p. 334).

These terms, already covered in the introductory chapter on functions, should be revisited, and students are expected to understand their relationship to the study of graphs (EXP.2). In addition, teachers are expected to negotiate these relationships formally, using mathematical language (EXP.3).

The analysis of the lesson text is presented below.

4.2 Lesson text

In this subsection, we seek to analyze the establishment of potential contract rules (as well as whether such rules remain in force in the lesson text or are broken at a certain point) and to what extent the expectations presented in the teacher's manual were met in the course of the knowledge approach.

The potential contract rules comprise regularities in the textbook's approach, which, due to their recurrence, will become part of a hypothetical teacher's habit, thus generating expectations in a hypothetical student regarding that teacher's actions (Eloi and Andrade, 2020). A total of four potential contract rules were identified in the analysis, one of which was explicit and the others implicit.

The concept of linear function graphs is presented in the textbook through the topic *Linear Function Graphs*$f(x) = ax + b$, which briefly discusses some of the implications of coefficients *a* and *b* in linear function graphs, as shown in the following excerpt.

> Geometrically, *b* is the ordinate at the point where the line, which is the graph of the function$f(x) = ax + b$, intersects the *Oy* axis, since for$x = 0$ we have$f(0) = a.0 + b = 0$. The number *a* is called the rate of change of the function *f*, but it is also known as the slope or angular coefficient of this line in relation to the Ox axis. The value *b* is called the initial value of the function *f* or the linear coefficient of this line (Dante, 2016, p. 79).

Thus, it is noted that the lesson text discusses that the value of coefficient *b* represents the ordinate of the point where the graph intersects the *Oy* axis; this coefficient is defined as the initial value. There is also a discussion about the coefficient *a*, which is defined as the rate of change, representing the slope of the line in relation to the Ox axis. It should be noted that the coefficients *a* and *b* are defined, respectively, as angular and linear coefficients[4].

This discussion converges with EXP.1, identified in the teacher's manual, in which work with the linear function graph should be based on exploring the implications of the Initial Value and the Rate of Change.

After the initial approach to the topic *Graph of the Linear Function*$f(x) = ax + b$, the

[4] In the context of the Potential Didactic Contract, the approach to coefficients *a* and *b* as angular and linear coefficients, respectively, unfolds new elements in the context of this contract, which, however, were not considered in this work.

subtopic *Plotting the Graph of Linear Functions* is presented, in which techniques for plotting the graph of the linear function are discussed through the presentation of graphs of seven different functions[5]. The following excerpt presents the introductory text to this subtopic and discusses how this graph is represented by a straight line.

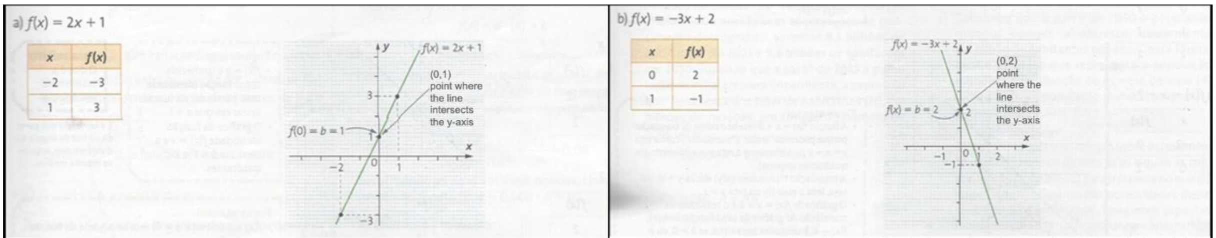
> *Let's construct the graphs of some linear functions f(x) = ax + b on the Cartesian plane.*
> *Since the graph of the linear function f(x) = ax + b is a straight line, and to draw a straight line we only need to know two distinct points belonging to it, we determine two distinct points of the function and draw the straight line.*

Figure2 : Excerpt from the textbook — Graph of the linear function (Dante, 2016, p. 79)

Thus, it is necessary to know two distinct points belonging to the line and plot the graph: "Since the graph of the linear function$f(x) = ax + b$ is a straight line and to plot a straight line we only need to know two distinct points belonging to it, we determine two distinct points of the function and plot the straight line" (Dante, 2016, p. 78). In light of the potential didactic contract, the above excerpt is configured as a potential explicit rule: to determine the graph of a linear function, it is necessary to determine two of its points (REP.1).

The potential explicit rules are like explicit mentions in textbooks that generalize procedures, creating a regularity regarding certain knowledge or activities (Eloi and Andrade, 2020). In the excerpt: "then we determine two distinct points of the function and draw the line" (Dante, 2016, p. 78), it is understood that a potential explicit rule is mirrored by the fact that it generalizes the procedure for constructing a graph, that is, to determine the graph of an affine function, it is necessary to determine two of its points.

The rest of the lesson text presents the examples of constructing the graph of an affine function already mentioned. The excerpt below shows the first two graphs.



In this case, we have a= 2 (a > 0), so the line is increasing (it "rises" from left to right). In this case, we have a = -3 (a < 0), so the line is decreasing (it "goes down" from left to right).

Figure 3 : Excerpt from the textbook — Exp. 1 and 2 on the graph of the linear function (Dante, 2016, p. 79)

In both cases, the discussion highlights the point of intersection of the graph with the Oy axis, i.e., the point $(0.b)$. This situation is consistent with EXP.1 in the teacher's manual, which states that the implications of the Initial Value and the Rate of Change should be explored when constructing the graph. In this sense, it is observed that these examples reflect possible classroom discussions about the point of intersection of the graph with the Oy axis, the relationship between the behavior of the graph and the sign of the coefficient a.

These examples also include a brief analysis of the direction of the graph, an evaluation of the value of the coefficient a, i.e., whether it is negative or positive, and the determination of the direction of the graph, whether it is an ascending or descending line. Following the examples, there are two cases of linear functions, the identity function and a case of the constant function. It should be noted that these functions are addressed in the literature as special cases of the affine function.

The textbook defines the linear function as a function presented in the form$f\ (x) = ax$,

[5] Within the scope of this analysis, only the first six graphs presented will be considered, given that the last example does not provide elements relevant to the analysis. Taking into account the maximum length for the text of this paper, we chose to omit it.

where the graph is represented by a non-vertical line that intersects the origin. At the same time, the identity function is considered a special case of the linear function, when the value of the coefficient a is equal to 1, in which the graph of this function is defined as the bisector of the 1st and 3rd quadrants. These definitions can be seen in the following excerpt.

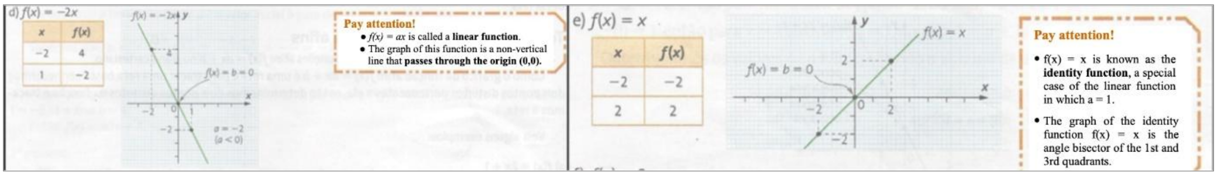


Figure4 : Excerpt from the textbook — Exp.3 and 4 on the graph of the affine function (Dante, 2016, p. 80)

The constant function is treated as the case in which the value of the coefficient a is equal to 0, so$f\ (x) = b$. The graph of this function is a straight line parallel to the Ox axis, and its image is the value of the coefficient b.

As discussed earlier, the lesson text contains the explicit rule REP.1, in which, to determine the linear function, the values of two of its points must be calculated. This rule is reinforced in the first five examples, i.e., only two points are used to construct the graph of the functions. However, in the excerpt below, it is possible to observe that the graph of the constant function is obtained using five points of the function, which breaks with REP.1.

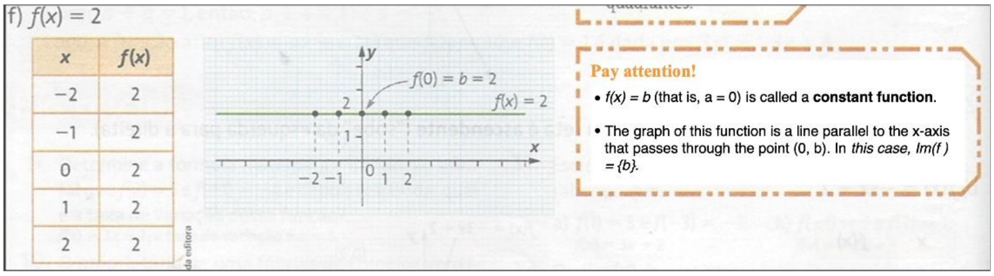


Figure5 : Excerpt from the textbook — Exp. 5 and 6 on the graph of the linear function (Dante, 2016, p. 80).

The moments of breaking with regularities, with potential rules imposed by the textbook, constitute evidence of ruptures (Eloi and Andrade, 2020). In this sense, it is believed that the above example is evidence of a rupture, as it does not comply with a potential rule proposed by the textbook (IRP.1).

From the analysis of the examples, it is possible to identify some regularities regarding the construction of the graph, which unfolded into potential implicit rules. For Eloi and Andrade (2020), potential implicit rules are regularities present in the textbook's approach that are not explicitly mentioned.

The first regularity identified concerns the values assigned to variable x for determining the points of the function. It can be observed that these values, in all examples, always belong to the interval between -2 and 2. Therefore, through this regularity, the following potential implicit rule should be stated: to plot the graph of the linear function, the values assigned to variable x range from -2 to 2 (RIP.1).

Another regularity is that, in all graphs, the point of intersection of the graph with the *Oy* axis is marked. There is always an indication that this value is obtained when the variable x assumes the value 0, which implies, by definition, that y assumes the value of the coefficient b. This regularity gives rise to two reflections: the first is linked to the expectation identified in the teacher's manual, in which the analysis of the initial value in the graph of the linear function should be explored. In this sense, the appearance of the initial value expresses the expectation that the teacher will negotiate that this value indicates the point of intersection with the graph.

On the other hand, this regularity can unfold into a potential implicit rule, in which, in the graphs of the linear function, there must always be a mark at the point of intersection with the *Oy* axis, and this point, in turn, has the *x-coordinate* equal to 0 and the *y-coordinate* equal to *b* (RIP.2).

It can be seen that all examples related to the construction of the graph of the linear function appear in their algebraic form, that is, the formation law is given and the graph is requested to be drawn, without exploring contextualized situations. In this sense, we believe that a potential implicit rule emerges in which exercises related to graph construction are always expressed in algebraic form, that is, the formation law is presented and the construction of the graph is requested (RIP.3).

4.3 Exercises

In this topic, exercises related to the construction of the graph of the linear function are analyzed. We sought to investigate the extent to which the expectations identified in the teacher's manual and the rules established in the lesson text are fulfilled in the exercises. A total of three exercises involving the construction of the graph of the linear function were identified (in the three questions, the construction of a total of ten graphs is requested).

The first one (Figure 5) asks for the construction of the graph of four functions. This exercise follows the pattern established in the lesson text, that is, the law of formation of the linear function is presented and the construction of the graph is requested.

13 In your notebook, construct a system of orthogonal axes and graph the following functions:

a) $f(x) = 2x + 3$ c) $f(x) = -2x + 5$

b) $f(x) = x + 3$ d) $f(x) = -2x - 2$

Figure 6: Excerpt from the textbook — Ex.1 on the graph of the linear function (Dante, 2016, p. 77)

In the second exercise (Figure 6), five linear functions are given, and students are asked to observe the influence of the rate of change on the construction of the graph. It should be noted that this exercise meets expectation EXP.1, identified in the teacher's manual, which emphasizes the importance of working on the influence of the function's coefficients on the construction of the graph.

14 In your notebook, using the same system of orthogonal axes, construct the graphs of the following functions. Then observe the influence of the rate of change on the position of each line. Is there any pattern to be noted?:

a) $f(x) = \frac{1}{2}x$ c) $h(x) = 2x$ e) $t(x) = -2x$

b) $g(x) = x$ d) $s(x) = -x$

Figure 7: Excerpt from the textbook — Ex.2 on the graph of the linear function (Dante, 2016, p. 77)

This exercise presents a situation beyond those covered in the lesson text, as it proposes not only the construction of function graphs, but also their comparison. Although this exercise does not constitute a break with the established rules, it is possible that, when worked on in the classroom, it may encourage new negotiations, as it presents new situations.

According to the teacher's manual, these exercises are fundamental for noting the relationship between the coefficient a and the graph of the linear function. In this scenario, it is understood that this exercise is permeated by the expectation that the implications of the Rate of Change in the construction of the graph will be explored.

The last exercise presents a contextualized situation about the trajectory of a moving object. In it, the formation law that relates the quantities of space and time is provided, and the construction of the corresponding graph is requested. This exercise breaks with RIP.3, identified in the lesson text, which states that graph construction activities are always presented in algebraic form.

> **15** *Physics*
> A body moves at a constant speed according to the mathematical formula s = 2t − 3, where s indicates the position of the body (in meters) at time t (in seconds). Draw a graph of s as a function of t in your notebook.

Figure 8: Excerpt from the textbook — Ex.3 on the graph of the linear function (Dante, 2016, p. 77)

Thus, it is understood that this exercise constitutes an indication of a break (IRP.2), as its nature contradicts RIP.3 by requiring the construction of the graph of the linear function from a contextualized situation.

It should be noted that the exercises analyzed go beyond the examples discussed in the lesson text, which explores examples with an explanation of the laws of formation and requests the construction of the graph. In the exercises, however, there are situations in which the behavior of different functions and the graphical interpretation of a contextualized situation must be compared.

Regarding the implicit rules related to the construction of the graph, it should be noted that it was not possible to ascertain, through the analysis of the exercises, whether they followed or broke these rules. These rules govern the procedures for constructing the graph, and it is not possible to observe such procedures in the wording of the exercises.

Table 3 summarizes the elements of the potential didactic contract identified in the course of this analysis.

Table 3: Summary of the elements of the potential didactic contract

Code	Description
	Expectation
EXP.1	Explore the influences of the Rate of Change and Initial Value in constructing the graph of the linear function, i.e., discuss that the rate of change is strictly related to the slope of the line representing the graph and that the initial value determines the ordinate of the point of intersection with the *Oy* axis.
EXP.2	Verify understanding of the terms *Ascending Line, Descending Line, Identity Function*, and *Constant Function* by working with graphs.
EXP.3	The teacher should discuss the growth and decay of a function and also discuss that these relationships should be expressed formally, using mathematical language.
	Potential Explicit Rules
REP.1	To determine the graph of an affine function, it is necessary to determine two of its points.
	Potential Implicit Rules
RIP.1	To plot the graph of the linear function, the values assigned to the variable x range from -2

	to 2.
RIP.2	The point of intersection with the *Oy* axis must always be marked, and this point, in turn, has an *x-coordinate* equal to 0 and a *y-coordinate* equal to *b*.
RIP.3	Exercises related to graph construction are always expressed in algebraic form.
Signs of Discontinuities	
IRP.1	Evidence of a break in the REP.1 rule caused by the situation in which the graph of the Constant Function is discussed.
IRP.2	Evidence of a breach of the RIP. 3 rules caused by the third exercise analyzed referring to the graph of the linear function.

Source: Own elaboration

5 Final considerations

The objective of this study was to analyze the potential didactic contract through expectations, potential contract rules, and evidence of breaches present in a 1st-year high school textbook regarding the graphical knowledge of the linear function. The choice of this knowledge was justified in part by the relevance of the linear function to the Basic Education curriculum, while research such as that by Santos (2002), Dornelas (2007), Delgado (2010), Santana (2016), Tozo and Oliveira (2016) specifically point out the difficulties surrounding its learning.

The term potential didactic contract was proposed in Eloi and Andrade (2020) and refers to a possible didactic contract present in the textbook's approach. These authors defined the elements of a potential didactic contract, which refer to a reinterpretation of the elements of a didactic contract already defined in the literature in light of the textbook.

The methodological procedures were structured through a documentary analysis of the textbook, and the analysis criteria correspond to the elements of the potential didactic contract.

Aiming to answer the question posed in the initial section of the article — How is the potential didactic contract established in the approach to the linear function graph in a 1st grade High School textbook? — it can be observed that this contract is established based on nine contractual elements, which correspond to three expectations, one potential explicit rule, three potential implicit rules, and two indications of ruptures. Among the expectations identified, EXP.1 stands out, which concerns exploring the concepts of rate of change and initial value in the construction of the graph of the linear function. This expectation seems to guide the approach to this concept, given that there are mentions in the lesson text and exercises that meet this expectation.

The only explicit rule, REP.1, states that in order to plot the graph of the linear function, it is necessary to determine two of its points. This rule is reinforced in five of the six examples analyzed in the lesson text. However, in the sixth example, there is evidence of a break from this rule (IRP.1), since five points are determined when plotting the graph of the linear function, rather than just two.

Three potential implicit rules were identified. The first (IRP.1) establishes that, in order to plot the graph of the linear function, the values assigned to the variable *x* must range from -2 to 2. The second (RIP.2) indicates that the point of intersection with the *Oy* axis must always be marked, this point being characterized by the coordinates of *x* equal to 0 and the coordinate of *y* equal to *b*. The third (RIP.3) consists of the fact that the exercises related to the construction of the graph are always expressed in algebraic form (RIP.3).

With regard to RIP., this rule is observed throughout the lesson text. However, as it refers to a construction procedure, it was not possible to verify the validity of this rule in the exercises. RIP.2 is consistent with EXP.1 because it refers to the construction of the graph, emphasizing the initial values and the rate of change. Regarding RIP.3, the rule is observed throughout the lesson text; however, this rule is broken in one of the exercises, generating IRP.2.

It is considered that the analysis of the regularities corresponding to the potential contract rules, especially the implicit ones, revealed marked didactic aspects regarding the approach to the graphic knowledge of the affine function. These regularities indicate a tendency toward standardization of graph construction procedures. This finding highlights the need to reflect on how textbooks can limit or favor the construction of broader mathematical meanings.

Returning to the study presented in Eloi (2019), it is observed that some of the potential contract rules identified, such as RIP.1, are also observed in the practice of the teacher using the textbook, indicating similarities between the potential didactic contract and the contract actually established in the classroom. This situation reinforces the need for further research that deepens the analysis of the potential didactic contract in the teaching of affine functions. In this sense, the studies presented in Eloi and Andrade (2022, 2025) stand out, as they have been contributing to this field by exploring the different contractual elements that permeate the approach to this knowledge.

Conflicts of Interest

The author declares that there are no conflicts of interest that could influence the results of the research presented in this article.

Data Availability Declaration

The data collected and analyzed in this article will be made available to the author upon request.

Note

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