10.46551/emd.v9n16a30 ISSN 2526-6136

Educação Matemática Debate

Origin and decline of the scientific discipline Descriptive Geometry

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Received • 01/04/2025
Accepted • 12/06/2025
Published • 13/12/2025

Editor • Rieuse Lopes

DOSSIER — HISTORY OF MATHEMATICS AND CULTURE

Abstract: Descriptive Geometry originated with the creation of *the École Centrale des Travaux Publics,* later renamed *École Polytechnique*. Two arts — Architecture and Fortification — were taken as teaching subjects, giving rise to this scientific discipline. The study aimed to analyze the process of rise, transformation, and decline of Descriptive Geometry, with a qualitative character, through the methodology of textual and contextual analysis. The *École Polytechnique* suffered a severe crisis between modernist and empiricist engineers, separating theoretical and practical teaching. Thus, the analyses reveal that the decline of this discipline occurred approximately fifteen years after its creation, with a complexity of social, cultural, epistemological, and corporatist factors and dimensions causing this decline.

Keywords: Descriptive Geometry. Stereotomy. Fortifications. Disciplinarization.

Origen y decadencia de la disciplina científica Geometría Descriptiva

Resumen: La Geometría Descriptiva tiene su origen en la creación de la *École Centrale des Travaux Publics*, futura *École Polytechnique*. Son dos las artes — Arquitectura y Fortificación — tomadas como objetos de enseñanza que darán origen a esta disciplina científica. El estudio tuvo como objetivo analizar el proceso de ascenso, transformación y caída de la Geometría Descriptiva, con carácter cualitativo, a través de la metodología de análisis textual y contextual. Sin embargo, la *École Polytechnique* sufrió una fuerte crisis entre los ingenieros modernistas y empiristas, separando la enseñanza teórica de la práctica. Así, los análisis revelan que el declive de esta disciplina se produjo aproximadamente quince años después de su creación, siendo una complejidad de factores y dimensiones sociales, culturales, epistemológicas y corporativistas las que provocaron dicho declive.

Palabras clave: Geometría Descriptiva. Estereotomía. Fortificaciones. Disciplinarización.

Origem e queda da disciplina científica Geometria Descritiva

Resumo: A Geometria Descritiva tem sua origem com a criação da *École Centrale des Travaux Publics,* futura *École Polytechnique*. São duas artes — Arquitetura e a Fortificação — tomadas como objeto de ensino que darão origem a essa disciplina científica. O estudo teve por objetivo analisar o processo de ascensão, transformação e queda da Geometria Descritiva, com caráter qualitativo, por meio da metodologia de análise textual e de contexto. A *École Polytechnique* sofre forte crise entre engenheiros modernistas e empiristas, separando o ensino teórico e prático. Assim, as análises revelam que a queda dessa disciplina ocorreu aproximadamente quinze anos após a sua criação, tendo uma complexidade de fatores e dimensões sociais, culturais, epistemológicas e corporativistas como causadoras dessa queda.

Palavras-chave: Geometria Descritiva. Estereotomia. Fortificações. Disciplinarização.

1 Introduction

The notion of scientific discipline belongs to the Sociology of Science and, particularly

Mathematics Education Debate
v. 9, n. 16, p. 1-16, 2025

in Brasil, there is a very different understanding of a higher education discipline[1] . Until the 18th century, science was not professionalized, and scientists had to work in various areas of research, that is, they were not specialized. This professionalization only happened with the emergence of public higher education systems and, with that, there is a new standard of practice that combines research and teaching, thus constituting the character of scientific discipline, institutionalized in higher education. As such institutions belong to the state education system, they are directly influenced by the dominant sociocultural values in the country.

Descriptive Geometry is, therefore, a scientific discipline that emerged almost suddenly, reaching a remarkable peak, admired, but then declining in various dimensions and degrees, and yet little noticed. Sociological studies show that disciplines do not always have continuity or develop successively, as many historians tend to assume without question. They can even decline, with movements in other directions, revealing other areas of research and teaching.

Thus, an analysis of Descriptive Geometry must be understood from its origins to its decline. The decline of Descriptive Geometry has not been understood or researched until now, and therefore, conceiving and revealing this has become the subject of research by the first author of this paper, with guidance from the second author. This unresolved situation of decline in historiography was observed mainly in papers such as that by Sakarovitch (1992), in which he promotes the idea, using verbs in the past tense, that Descriptive Geometry achieved the objectives proposed at its origin, avoiding addressing the decline of this science. However, he presents this decline quite late. In contrast, Coolidge (1940) points to the decline of Descriptive Geometry shortly after its creation, claiming that there were no advances after the publications of Jean Nicolas Pierre Hachette (1769-1834).

[2]This made it possible to identify two axes of research: i) the heyday of the discipline of Descriptive Geometry — in terms of its important role in the formation of *the École Polytechnique,* combining theory and practice; and ii) its decline — when it became a repetition of methods disconnected from its basic arts, stereotomy, and fortifications. Thus, it became necessary to delve deeper into the origins of Descriptive Geometry and its rise as a scientific discipline. The search for its origins proved to be quite complex and revealing, since traditional historiography presented its origins from a unilateral dimension, that is, only Architecture, disregarding the studies and problems of *défilement* and fortifications. In addition, misleading statements claimed, for example, the use of Euclid's Geometry in Architectural projects in the Middle Ages. It was thus understood that these two movements and developments of different social and professional groups, in the Fine Arts and the Martial Arts, were important in the process of conceptualizing Descriptive Geometry and that they were articulated for the first time in an educational institution for training engineers — the *École Royale du Génie de Mézières*, since 1748.

Another revealing point of the research, and one that justifies the analysis since Antiquity, is the meaning given to the concept of one of the fundamental elements of Vitruvius' Architecture, solidity *(firmitas)*, which, in the opinion of the authors of this paper, will constitute the main reason for the *mathematization* of construction processes and, consequently, of the process of training engineers. This solidity will undergo changes in its conception over the centuries, which will substantially alter attitudes and, consequently, the conceptions of Architecture and Fortification, which are the basic elements of Descriptive Geometry, according to the conception of the authors of this paper.

Thus, two great arts, Architecture and Fortification, will constitute the structuring

[1] This paper is an expanded version of the paper published in the proceedings of the IX International Seminar on Research in Mathematics Education.
[2] *Défilement* is the *art* of protecting points within the perimeter of a fortification from enemy fire from the surrounding space outside the fortification (Carlevaris, 2014, p. 632)

elements that, when placed as objects of teaching, merge and create a science: Descriptive Geometry. In this way, the *École Royale du Génie de Mézières* will be the ideal environment for this union. The Enlightenment concepts of the French Revolution will make this one important instrument of change. Linked to the French Revolution, the *École Polytechnique* is born, which will become the stage for the rise, but also the fall, of this discipline; in addition to influencing higher education in several countries, including Brazil.

The approach includes a careful analysis of documents, seeking subjects that are either well researched or not, in the literature, based on an intensive search of primary sources in order to clarify open questions and analyze developments, from a methodologically reflective perspective, to go beyond the myth with which the history of Descriptive Geometry is normally evaluated.

The research was developed based on one main question: what are the conceptual, sociocultural, and political influences on the rise, transformation, and decline of Descriptive Geometry? To answer this question, the general objective of the research was established: to analyze how Descriptive Geometry rose, transformed, and declined. To this end, the following specific objectives were established: (i) to investigate works that address Descriptive Geometry, modern architectural treatises, and the influences on the work of master masons in the medieval period; (ii) to investigate teaching at *the École Royale du Génie de Mézières*; (iii) to deepen the conceptual understanding between theory and practice; (iv) to investigate the institutional development that occurred in France.

2 Methodological aspects and state of the art

As a human and social activity, research inevitably carries values and preferences that guide the researcher, since it is from the questions they ask themselves, based on what they know about the subject and the theory surrounding it, that knowledge about the researched fact is constructed (Lüdke and André, 2020). In this study, we opted for qualitative research, using the methodology of textual analysis, also understood as documentary analysis by Lüdke and André (2020), since this is qualitative research in science and education. Parallel to textual analysis, the research was based on historical analysis of context as a situation from a critical approach. The word context refers to the set of concrete gestures, procedures, and codified practices, attested to varying degrees by written sources, referred to as context as situation, according to Bernard and Proust (2014). Although context as situation can be applied generally to various objects of historical study, it is particularly useful and relevant in the case of teaching and learning contexts. More specifically, this methodology of analysis takes into account the contexts of transmission, notably the teaching context in which knowledge was developed, used, and transmitted (Bernard and Proust, 2014).

In particular, the analysis of the decline of Descriptive Geometry as a field of research and teaching took into account the context of the French Revolution and the conservatism of corporations. Thus, the textual production combined description and interpretation in an effort to express the understandings reached through intense immersion in the *corpus* of analysis. For Moraes and Galiazzi (2016, p. 167), “describing and interpreting, when conceived together, are part of the effort to express the understanding of a phenomenon”.

In addressing Descriptive Geometry in this work, we sought mathematical practices that explain this knowledge as a discipline and object of research. Therefore, based on the evidence, contexts, and relationships, the study aimed to answer the guiding question, which constitutes this work as research in the History of Mathematics based on three guiding dimensions of the research: (i) the conceptual understanding of the discipline; (ii) the changes undergone in its teaching process; and (iii) the process of disciplinarization in Brasil.

The analysis of the origin of Descriptive Geometry did not occur in an internalist and intramathematically manner, but rather based on a sociopolitical methodology of contextualization, corroborating with Taton (1954, p. 5), "the history of the origin of Descriptive Geometry cannot be simply attributed to the history of Geometry, but must largely be attributed to the techniques and history of Art". Therefore, the research was outlined by a bibliographic approach involving an extensive search of the literature of works already published on the history of Descriptive Geometry for careful evaluation and analysis of these texts; with the aim of i) analyzing the subjects that are well or poorly covered in the literature, highlighting contradictions and showing aspects that have not yet been considered; ii) searching for primary and secondary sources that are not yet known or considered; and iii) comparing these primary and secondary sources in search of new results.

The literature review aims to share the results of other studies closely related to the research, in order to enable reflection and broaden knowledge on the subject. Based on the literature, it appears that research on Descriptive Geometry is recent, as demonstrated by the studies of: (i) Évelyne Barbin — French science historian and professor at the University of Nantes: *Descriptive Geometry in France: History of élémentation of a method* (1795-1865) published in 2015 and *Monge's Descriptive Geometry: his Lessons and the Teaching Given by Lacroix and Hachette* published by Springer Editora in 2019; (ii) Joël Sakarovitch — professor of History of Science at René Descartes University (Paris V), paper published as a chapter in the book *Enciclopédie des métiers: la maçonnerie et la taille de Pierre*, which presents Girard Desargues' stone cutting booklet, published in 1640 and the book *Épures d'Architecture: de la coupe des pierres à la géométrie descriptive XVI*[and] *– XVII*[and] *siécles*. (iii) Gert Schubring, Vinicius Mendes, and Thiago Oliveira — *The dissemination of Descriptive Geometry in Latin America*; (iv) Danusa Chini Gani — professor of Architecture at the Federal University of Rio de Janeiro (UFRJ), with the dissertation entitled *The lessons of Gaspard Monge and the subsequent teaching of Descriptive Geometry,* defended in 2004, in the Postgraduate Program in Engineering at UFRJ.

When searching the thesis and dissertation database for *Descriptive Geometry* for a five-year period, nine theses and five dissertations were found. Of the nine theses, eight used digital technologies, and the five dissertations were part of professional master's degrees with applications to teaching. The only thesis with possible approximations — Douglas Gonçalves Leite's thesis, entitled *A Geometria de Gaspard Monge* [The Geometry of Gaspard Monge] — was not available in its complete version at the time.

It is worth noting that the literature review allowed us to identify some myths, such as the attribution of knowledge of Euclid's *Elements* to justify the geometric experiments of medieval master masons, as well as to reveal the one-dimensional view, based on Architecture, of the modes of representation of three-dimensional objects on a plane. The process of disciplining Descriptive Geometry ends up being looked at in Secondary Education, with little discussion in Higher Education. In addition, the review allowed us to recognize the absence of a look at the process of the rise of Descriptive Geometry, more focused on social practices applied not only by the Fine Arts (through Architecture), but also by the Martial Arts. We also observed a tendency to disregard existing movements in scientific disciplines, attributing continuity to them. Historiography tends to view the discipline as something eternal, that is, once it begins, it will continue to develop. In evaluating this literature, it was possible to identify several important dimensions that could be revealed.

3 Descriptive Geometry: from empeiria (experience) to episteme (science)

The search for the foundations of Descriptive Geometry suggests that the path of the evolution of the graphic representation of three-dimensional space must be traced. Therefore, investigating the originality in the development of geometric methods is fundamental. This search focuses on the processes of solving stereotomy problems. Thus, this Architecture is articulated with a more constructive Geometry, but with modernity, it is articulated with a Scientific Geometry.

3.1 The geometric knowledge of medieval architects and master masons

The search for the origin of Descriptive Geometry, a discipline created in the late 18th century, dates back to the Treatises on Architecture of the Modern Era. These treatises originated in the late 15th century with Humanist architects who revived concepts idealized in Antiquity.

This study identified that many treatises, of various kinds, were written in Antiquity with the purpose that all the knowledge built up until then could be used in posterity. However, these ideas were no longer recorded, or were rarely recorded, marking a period in which knowledge of the profession was passed down orally and restricted only to members of corporations, a characteristic that became consolidated in the Middle Ages. The question that arises is: at what point did the culture of texts through treatises disappear?

The breakdown of textual culture and, in certain respects, the breakdown of Greco-Roman culture can be marked by the fall of the Western Roman Empire in 476, as well as the constant barbarian invasions that plunged civilization into a period known as the *Dark Ages,* which lasted until approximately the reign of Charlemagne. The only known treatise on Architecture from antiquity is the work of Vitruvius, *De Architectura,* also known as the *Ten Books* of Vitruvius. This important work dates from the 1st century BCE[3], according to his own accounts. Vitruvius lived until the period ruled by the Roman emperor Caesar Augustus (27 BCE-14 CE). Little is known about Vitruvius, but his own text reveals some details about him, such as the fact that he did not belong to a wealthy family and that his parents took great care in his education and the instruction given by his teachers.

> Therefore, I am very grateful and infinitely thankful to my parents for the approval of this Athenian law and for taking care that I learned an art, and one that cannot be brought to perfection without learning and liberal education in all branches of instruction. Thanks, therefore, to my parents' attention and the instruction given by my teachers, I have acquired a wide range of knowledge, and through my enjoyment of literary and artistic subjects and the writing of treatises, I have acquired intellectual possessions whose main fruits are these thoughts: that superfluity is useless and that not lacking anything is true wealth. There may be some people, however, who consider all this inconsequential and think that wise men are those who have a lot of money. That is why many, in pursuit of this end, take on an impudent confidence and achieve notoriety and wealth at the same time (Morgan, 1914, p. 168-169).

Vitruvius' text became well known from Antiquity to the Renaissance, as it was copied into a considerable number of manuscripts. Of the 78 manuscripts of his text that currently exist, 34 date from before the 15th century (Vitorino, 2004). The return of the Vitruvian tradition of architectural treatises is marked by *De Re Ædificatoria,* or as it became better known, the *Treatise on Architecture* by Leon Battista Alberti (1404-1472). The text was written in 1452 and was the first work of its kind to be published in modern times, more precisely in Venice in 1485. This work was based on *De Architectura* and was organized according to the three basic and elementary principles of Architecture defined by Vitruvius: *firmitas, utilitas*,

[3] before the Common Era.

and *venustas,* respectively, solidity, consistency (utility), and beauty. Apart from Vitruvius' precious treatise, which dates back to the 1st century BCE, there is no personal testimony from any architect of Antiquity and very little from the Middle Ages. As a result, there are practically no texts on the transmission of ideas and skills during this period. Few drawings have survived, and it can be assumed that a great deal of discourse remains lost. Most of the literary evidence comes from the Middle Ages, from clerical sources consisting mainly of documents containing calculations relating to buildings or drawings by architects and masons (Rykwert, 1984). Masons (stone masons) and other craftsmen were always linked to a guild which, according to Rykwert (1984), consisted of a secret society in which the transmission of ideas took place within it and its procedures were deliberately not recorded.

It is thanks to Villard Honnecourt's 13th-century *Sketchbook* and the fifteen booklets written and printed by German master masons that, at the end of the 15th century and beginning of the 16th century, scholars on the subject were able to enter on firmer ground. Villard Honnecourt was an important French master mason of the 13th century. His *Sketchbook* contained problems applied to drawing faces and bodies of men and animals, as well as ways to calculate tower heights. One of these German master masons was the experienced Mathias Roriczer, who published a booklet on how to build pinnacles, the elevated parts of a building.

The research revealed the difficulty in designating and appointing architects in the Middle Ages. It was understood that architects were no longer those who were highly knowledgeable about the craft of construction, as this profession in the Middle Ages was associated with bishops, dukes, counts, and abbots who were responsible for the execution, from foundation to roof, of a royal church. According to Binding (1993), many of the references go back to the Bible, in Paul's first letter to the Corinthians, chapter 3, verse 10 (Corinthians 3:10). In it, Paul says: “By the grace of God given to me as a wise architect, I laid the foundation, but let others build on it”. Thus, in the 12th and 13th centuries, theologians, in particular, as donors, builders, and organizers of buildings, were repeatedly referred to as architects.

Literature focused on the Middle Ages also showed that most authors who studied the history of Architecture assumed a supposedly known Euclidean geometry. In fact, geometric practices were revealed to be much more based on the geometry practiced by Vitruvius, that is, on an empiricist practice. Therefore, the studies by Jens Høyrup (1987, 1990, 2014) on what he calls scientific and sub-scientific knowledge were taken as a basis. In short, scientific knowledge aims at truth and sub-scientific knowledge is directed towards utility. Thus, instead of claiming a use of the scientific geometry of Euclid's Elements in the Middle Ages, the practices of geometry in that period consisted of a knowledge of sub-Euclidean geometry, as called by Høyrup (2014) or also called constructive geometry by Shelby (1972).

Sub-Euclidean geometry would be that derived from surveyors and strata of Euclidean texts, but which contained Euclidean definitions, postulates, and propositions from the first three Books of Euclid (Høyrup, 2014). Constructive geometry consists of basic geometric procedures for transformation into architectural drawings (Shelby, 1972). Therefore, the transmission of knowledge followed a sub-scientific tradition (Høyrup, 1990), taking place within guilds or Bauhütte workshops in Germany, meaning that master masons did not receive formal education and traditions were transmitted orally. Guilds were trade corporations, associations of craftsmen from the same field, that is, people who carried out the same professional activity and sought to guarantee the interests of their class and regulate the profession. Bauhütte is a German term for institution/workshop, created in the Middle Ages by all workers on a construction site, mainly a cathedral.

3.2 The beginning of the theorization of Architecture: conceptual emergence of Descriptive Geometry

Stones were used as a building material due to their durability and availability. Geometric problems in Architecture arise due to the cutting of stones associated with the construction of vaults and domes. But vaults and domes have existed since ancient times. So what has changed?

It all starts with the improvement in stone cutting methods for the construction of arches and keystone vaults, i.e., those arches and vaults that contain a wedge-shaped central stone. This rests on the neighboring voussoirs, allowing the lateral forces between them to resist the forces received by the arch (Figure 1). However, when the scarcity of raw materials began to dictate the need for economy in construction processes, the need for more precise geometric methods arose.

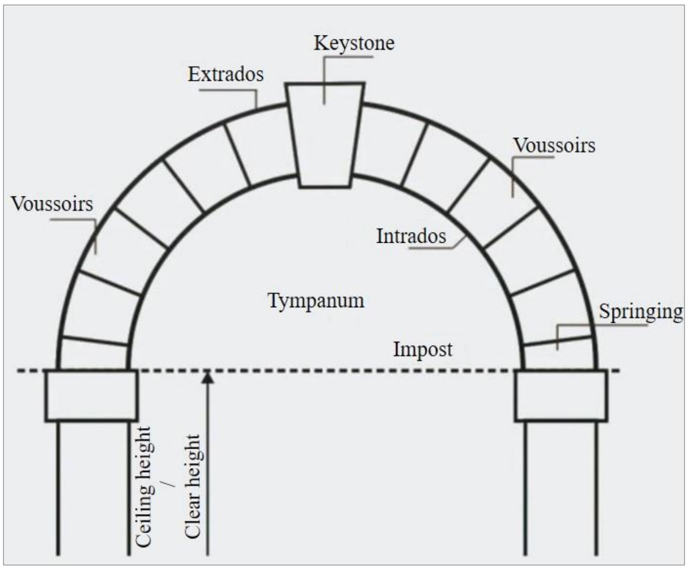

Figure 1: Keystones and voussoirs (https://contatoboxconstruc.wixsite.com/websitebox/post/a-engenharia-das-estruturas-em-arco; Accessed Jun. 25, 2024)

This marks the beginning of a change in thinking in construction, since, throughout Antiquity and the Early Middle Ages, buildings were designed in the order in which they would be built, i.e., from the bottom up. Changes in cutting techniques in the construction of vaults required a reversal in architectural thinking, and this became the first step towards the theorization of construction processes (Sakarovitch, 1998).

It is possible to observe, with this, the emergence of a fourth element to be added to the elementary principles of Architecture defined by Vitruvius, Economy. Thus, economy, together with stone cutting techniques for the construction of arches and keystone vaults, will be responsible for the beginning of theorization in Architecture. This is evident in the Treatises on Architecture of Modern Times, from the 16th century onwards, especially Philibert de l'Orme's Treatise on Architecture. Philibert de l'Orme's Treatise on Architecture (1514-1570), published in 1567 — *Le premier tome de l'archictecture* — contained two books (Books III and IV) entirely dedicated to stereotomy, that is, the technique of dividing, cutting, and precisely adjusting stones. Philibert de l'Orme's treatise marks the beginning of theorization, precisely because de l'Orme uses the study of projections to determine the true size of stones (Sanabria, 1989). Thus, in seeking methods to express exact measurements, Philibert de l'Orme inaugurates a new conception of treatises (Sakarovitch, 1998). It is, therefore, the use of stone cutting technology for the construction of keystone vaults, combined with the economy of raw materials — in this case, stones — that gave rise to scientific geometry, in Høyrup's conception (1987, 1990, 2014), in construction processes, referred to by the authors as *mathematization* in construction processes.

But it was in the 18th century, with the publication of volume 1 of Amédée Frézier's

(1682-1773) Treatise on Architecture, published in 1737 — *La théorie et la pratique de la coupe des pierres et des bois pour la construction des voûtes ou Traité de stéréotomie à l'usage de l'Architecture* —, that *stone cutting* came to mean much more than the mathematics underlying the cutting of stones. Volume 1 deals with the sections of round bodies, cones, cylinders, and spheres, as a way of understanding the parts and intersections of vaults. The importance of this treatise is that, in addition to the processes of three-dimensional space projection, it marks an attempt to generalize and theorize, defining the relationship between Empiricism and Rationalism in architectural processes. In addition, Frézier's treatise would influence Nicolas Chastillon (1699-1759) in his teaching of engineering at *the École Royale du Génie de Mézières.* It was the emergence of a new element, teaching, that would articulate Empiricism and Rationalism. As a result, the dominance of empiricism began to be replaced by teaching that linked theory and practice, promoting the development of educational institutions.

4 The rise and fall of Descriptive Geometry

The graphic techniques of stereotomy treatises already evidenced the principle of double projection and the use of auxiliary planes, especially in Frézier's treatises. But what distinguishes Frézier's Geometry from Monge's Descriptive Geometry is the objective of each of these geometries. While one uses geometric methods for greater efficiency on construction sites, the other is concerned with the pedagogical aspects that these methods provide in the training of engineers. Thus, understanding the principles of the teaching offered by the École du Génie de Mézières is fundamental.

4.1 The importance of *the École Royale du Génie de Mézières* for Descriptive Geometry

The *École Royale du Génie de Mézières* was founded in 1748 by Nicolas-François-Antoine de Chastillon (1699-1759). Two years after its creation, all future engineers had to attend the *École de Mézières* before obtaining their license. Thus, from 1751 to 1793, all students admitted to the engineering competition spent two years there before obtaining their engineering certificate. It quickly gained a great reputation, both for the quality of the students recruited and for the seriousness of the education provided there (Taton, 1951).

Teaching at *the École Royale du Génie de Mézières* was divided into two stages: the first presented the fundamentals and the second presented the applications. The fundamentals included the teaching of arithmetic, geometry, static mechanics, and hydraulics, and the applications consisted of drawing and model making for stone and wood cutting, as well as topographic surveys and siege and defense exercises, i.e., Architecture and fortification. This fundamentals-application structure continued at *the École Royale du Génie de Mézières and was repeated* at *the École Polytechnique* (Sakarovitch, 1998).

The *École Royale du Génie de Mézières* operated from 1748 to 1794, when it was transferred to Metz, with a greatly reduced function. Created to train the entire corps of engineers in France, its places were reserved for young people of noble origin. However, in the 1760s, Chastillon created an institution attached to the officers' school with the aim of training technical personnel to work with the engineering corps, mockingly called *La Gache* (Sakarovitch, 1998; Arago, 1854). *La Gache* students were not bound by wealth or birth, but rather by their abilities, and could therefore apply for the position of second lieutenant of engineers (Arago, 1854).

Chastillon saw stereotomy as a way of developing spatial vision and providing training in geometry. Therefore, stereotomy is no longer understood as a construction technique but rather as the genesis of geometric theory (Sakarovitch, 1998). Another art was taken up as a subject of instruction at *the École Royale du Génie de Mézières*: fortification. The 18th century was a turning point in approaches to warfare. Weapons and their range were improved, and fortifications became increasingly complex structures. As a result, *défilement* became a mandatory subject of study for defensive fortification. Cannons that could fire up to 1,400 meters and other devices appeared on the battlefield. The goal was to raise the weapons and attack the fortification from above (Carlevaris, 2014). At this point, the study of *défilement* became more similar to the study of geometry. However, until the mid-18th century, *défilement* problems were still treated empirically by military engineers who followed in the footsteps of Sebastian de Vauban (1633-1707), and Louis de Cormontaigne (1696-1752). Therefore, it is these two arts, Architecture and Fortification, that will be taken as the object of teaching. The first *mathematized* and the second still empirical. This union will be, in the authors' conception, the first step towards teaching what will become Descriptive Geometry.

However, it was with the arrival of Gaspard Monge (1746-1818) at *the École de Mézières* at the end of 1764 that he solved the problem of *défilement* in 1765 using purely geometric methods (Figure 2). However, this method was kept secret and, therefore, was not part of the teaching at Mézières. Figure 2 shows the solution to a *défilement* problem carried out by Monge, using the principles of projections that were already used in stereotomy.

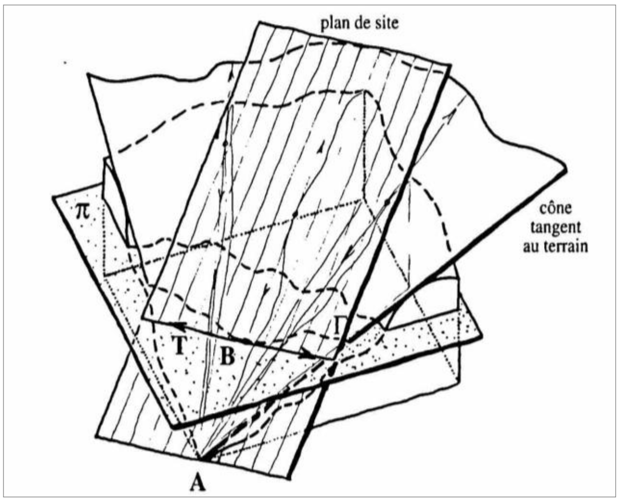

Figure 2: Monge's geometric solution to the *défilement* problem (Sakarovitch, 1998, p. 228)

Gaspard Monge arrived in Mézières at the invitation of the senior engineering officer, Colonel Antoine-Nicolas-Bernard du Vignau, after presenting the design of the plan of his hometown, Beaune (Arago, 1854). As Monge was not of noble origin, he was assigned to *La Gache*. At *the École*, Nicolas Pierre Hachette was a student of Monge's before becoming his assistant at *the École Normale* and the *École Polytechnique* (Sakarovitch, 1998). By proposing a graphic technique that replaced empirical attempts, Monge was rewarded for his invention with the position of tutor in mathematics, that is, he gave individual lessons to students in study rooms suitable for graphic work. But it was not until 1783 that he took over as professor of mathematics, replacing Bossut (Arago, 1854).

4.2 The heyday of Descriptive Geometry

Descriptive Geometry emerged as a scientific discipline suddenly and at a unique moment in history: in the early years of the French Revolution, establishing higher education in the first public education system. In this context, it proved to be paradigmatic for the value of science for practice, as postulated by the Enlightenment.

On this subject, Monge highlighted this absolutely new and innovative character in his *Programme* (1795), with which he introduced his lessons in Descriptive Geometry at *the École Normale de l'an III*, emphasizing the teaching of science as a means for the economic development of the nation. He also highlighted, in particular, the role of Descriptive Geometry in new learning practices and national industry. In fact, the *Programme* was implemented, placing Descriptive Geometry as a major scientific discipline in public higher education, now established for the first time in a country.

In this role, Descriptive Geometry became one of the main disciplines at *the École Centrale des Travaux Publics*, created in 1794 as a new type of higher education focused on applications. Renamed *École Polytechnique* a year later, the focus on professional training also marked, even more paradigmatically, the articulation of theory with practice. Along with chemistry, Descriptive Geometry was a key representative of the method of analysis, understood as an epistemological means of acquiring scientific knowledge, as well as, at *the École Normale de l'an III*, 1795, for the training of teachers — for the new public education system.

The analysis of this period in the history of Descriptive Geometry is based heavily on a source that is little used in the extensive literature on Descriptive Geometry: the *Développemens sur l'enseignement adopté pour l'École Centrale des Travaux Publics*, prepared by Monge in 1794, which constitutes the first curriculum for a public higher education institution. Furthermore, it is the first time that the term Descriptive Geometry appears formally in a document, in which Monge even explains what it means. In this document, the author states that the objective of *the École* is to provide young people with the knowledge necessary to organize, direct, and manage all types of work, commanded for general utility and executed at the expense of the Republic.

Développemens sur l'enseignement de l'école centrale des travaux publics presents Descriptive Geometry as a language that should become habitual and, therefore, students should practice it continuously during the three years of the course at *the École* (Monge, 1794, p. 4). According to Monge (1794),

> Descriptive Geometry is a necessary language common to engineers who design projects, artists who must direct their execution, and workers who must carry them out. This language, which lends itself to precision, also has the advantage of being a means of searching for truth and achieving desired and unknown results. Like all other languages, it can only become familiar through habitual use; thus, during the three years of instruction at *the École Centrale des Travaux Publics,* students will practice it continuously. (p. 4)

Descriptive Geometry also has a second function as a scientific language, as it is considered a language susceptible to precision which, according to Condillac (1715-1780), is a method of research (Paul, 1980). Although originally designed for the *École Centrale des Travaux Publics,* Descriptive Geometry was first taught at *the École Normale de l'an III,* whose objective was to instruct future teachers in the useful sciences and the art of teaching them. The courses at *the École Normale* lasted four months and were attended by around 1,400 students, citizens from all parts of the Republic who were already educated in the sciences, who would be responsible for training teachers to work in primary schools in various regions of France.

However, it was at the *École Polytechnique* that Descriptive Geometry would experience its heyday and also its decline. The two arts, Architecture and Fortification, which had already been taken up as subjects of instruction at *the École Royale du Génie de Mézières,* the former being *mathematized* and the latter still very empirical based on the methods of Vauban and Cormontaigne, would form the foundations, or pillars, of this new discipline. Unlike the *École Royale du Génie de Mézières*, these two arts were already *mathematized* from the outset, that is, both Stereotomy and Fortifications already used purely geometric processes. This was only possible after the French Revolution, marking a break with practice-centered teaching. However, the *École Polytechnique* had to deal with the coexistence of two types of engineers that marked the beginning of the 19th century. On the one hand, there were the *artist* engineers, or, that is, the empiricist engineers trained in the last years of *the Ancien Régime*, and on the other, the modernist engineers, that is, those who had attended the *École Polytechnique* and acquired a more extensive scientific culture.

4.3 The decline of Descriptive Geometry

The union of the two arts, Architecture and Fortification, in the mathematical conception of Descriptive Geometry, met with resistance within the military engineering faculty itself. Furthermore, since the change of name — and, consequently, of concept, when it ceased to be *the École Centrale des Travaux Publics* to become *the École Polytechnique,* one year after its creation — the institution became a basic training school for the *Écoles d'Application.*

The 1795 law*,* although it seriously altered the institutional context of *the École Polytechnique,* did not provide for operational regulations governing the relationship between the application schools and the basic training school. This lack of concrete measures between these institutions led to the first crisis, which began in November 1796 (Schubring, 2004). As a result, practical activities were to take place at *the écoles d'application*, leaving more general knowledge to the *École Polytechnique*. One of these schools was the *École du Génie de Metz*, where the corps of engineers, trained under *the Ancien Régime*, attempted to reestablish an independent function, like the former *École de Mézières*. However, it was only with the law of 1799 that this relationship became well established. Thus, since 1800, the concept of integrated training has undergone disruptions, reaching its peak in 1810.

The changes undergone in the successive teaching programs of *the École Polytechnique* showed that the combination of Descriptive Geometry and Physics/Chemistry in its original teaching program gave way to the combination of Analysis and Mechanics. However, Mechanics remained a separate subject from Analysis, since the original conception of analysis, in the sense of Condillac's analytical methods, gave way, as of the 1799 legislation, to Analysis as a mathematical discipline — Differential and Integral Calculus — thus using the same term to represent something totally different.

As a result, the discipline of Descriptive Geometry underwent reductions in favor of Analysis, Differential and Integral Calculus, and Mechanics (Fourcy, 1828). Thus, in 1806, Analysis already assumed the role of global leadership among the teaching disciplines at *the École Polytechnique* (Schubring, 2004).

With the organization of Metz's curriculum in 1807, the disruptive element — basic knowledge of Mathematics and Science — was eliminated from the training system, and the corporatism in action of *the Corps du Génie* was limited (Schubring, 2004). It was at this point that the subject of Fortification ceased to be taught at *the École Polytechnique* and became a special course for the specific field of engineering. Schubring (2004) attributes this achievement to *the commission mixte* and, in particular, to Allent. However, it is important to emphasize the need to analyze both sides of this struggle. Although this decision was an achievement for the *École* de Metz, it constituted a defeat for the *École Polytechnique* and the entire initial concept of its training integrating theory and application. This stance represents a break with Descriptive Geometry and its foundations — Architecture and Fortification.

Furthermore, the teaching of Fortification continues at *the École de Metz* with a return to traditionalist processes — based on the methods adopted by Vauban and Cormontaigne — against modernist processes based on Mathematics and scientific knowledge. This shows that Fortification, as the foundation of Descriptive Geometry, suffered a rupture in the 1810 program.

Thus, it can be observed that Architecture became increasingly *mathematized* in the concepts of Analysis, Differential and Integral Calculus, and Fortification focused on traditional empiricist methods.

It is thus established that the decline of Descriptive Geometry in its initial conception occurred about fifteen years after its creation, since its basic principles were completely dismantled. What remains is a repetition of procedures completely disconnected from practice, with the sole purpose of developing spatial visualization skills, that is, a return to the objective of teaching stereotomy at *the École du Génie de Mézières.*

5 Final Considerations

The final considerations are presented based on textual and contextual analyses from the research *corpus*. The search for the decline of Descriptive Geometry allowed for an understanding of the process of rise, transformation, and subsequent decline of Descriptive Geometry — the decline of a scientific discipline has been discussed in works such as those by Bos (1984) and Noble (2022), showing that mathematics does not develop in a linear fashion. The search for the origin of this scientific discipline led the authors to the architectural treatises of Modern Times, as the literature suggested. However, immersion in the geometric procedures used in construction processes allowed for the analysis, since Antiquity, of the only architectural treatise that has survived to the present day — the *Ten Books* of Vitruvius.

Immersion in Vitruvius' treatise enabled us to understand the concept of Architecture in Antiquity and the geometry used by him. Although geometric procedures were used, they were characterized by constructive processes based on sequences of procedures, even when addressing elements of geometry. In addition, it was possible to identify the basic and elementary principles of Architecture defined by Vitruvius, which still underpin this art form today: *firmitas* (solidity), *utilitas* (utility), and *venustas* (beauty). Throughout the studies, it was identified that the solidity of Antiquity would give way to a new concept in Modernity, which led to new attitudes.

Based on the analysis of the geometric knowledge of Antiquity, from the work of Vitruvius, a large gap in the study of the Middle Ages became evident. Due to the culture of orality and the lack of written records, identifying the geometric knowledge of medieval master masons was a great challenge. Furthermore, there was a major obstacle to understanding the concept of the architect in the Middle Ages, since this designation was associated with divinity. Thus, studies have identified that medieval architects were, in fact, abbots and bishops responsible for construction during that period, usually religious works, and therefore, architects, in Vitruvius' conception, were the so-called medieval master masons.

From immersion in *the corpus* of analysis, the emergence of a new constructive principle was observed, economy, which dictated, and has dictated, together with the principle of the solidity of modernity, the process of *mathematization* of constructive processes.

Thus, from Antiquity to the first Renaissance Treatise on Architecture, by Leon Battista Alberti, the Geometry present translates into a knowledge of sub-scientific Mathematics, that is, sub-Euclidean Geometry, also known as constructive geometry. Thus, it is only from Philibert de l'Orme's Treatise on Architecture that we can affirm the beginning of scientific geometry, that is, the use of scientific mathematics in the application of the solidity of modernity associated with the principle of economy. However, until the early 18th century, these treatises consisted of tools for construction, seeking a language specific to builders and those who would direct the works, except for Amédée Frézier's Treatise, which was specifically aimed at engineers.

Solidity, as a principle of Architecture and traditionally identified since ancient times

— the need to build for eternity — referred to by the authors of this work as traditional solidity, revealed itself through a new understanding — called by the authors the solidity of modernity — which characterized a change in thinking among 18th-century builders. This led to the emergence of more skilled builders, whose art of engineering became increasingly dependent on the mathematical concepts underlying construction processes. It is, therefore, according to these analyses, that it is concluded that the solidity of modernity and economy are the new principles that dictated, and still dictate, the *mathematization* in the training of engineers from the 18th century onwards.

The mathematics underlying stone cutting, already present in architectural treatises such as that of Amédée Frézier, would form the basis of the work at *the École du Génie de Mézières*, the cradle of descriptive geometry. It is, therefore, based on Amédée Frézier's treatise that stereotomy began to be treated much more as the mathematics underlying stone cutting, whose conception inspired Chastillon on the teaching of stereotomy at *the École Royale du Génie de Mézières*. The analysis of Chastillon's work was only possible from writings, such as memoirs, in Théodore Olivier's work, published in 1847. However, the figures were not part of Olivier's work and, surprisingly, were found in a German dissertation. And even though they were discussed in Belhoste, Picon, and Sakarovitch (1990), the work did not include the figures.

Thus, it was in research on the teaching of fortifications and the study of *défilement* at the *École du Génie de Mézières* that the important use of projections of three-dimensional objects on a plane for the exclusive use of the military service was identified. Thus, contrary to what traditional historiography suggests, the two great arts, Architecture and Fortification, were taken as the basic arts that gave rise to the discipline of Descriptive Geometry.

The *École du Génie de Mézières* played an important role in this process precisely because it was the ideal environment for bringing together the two arts, Fine Arts and Military Arts, turning them into teaching tools. In this way, the study of three-dimensional geometric projections, already covered in the first volume of Frézier, was combined with applications in the teaching of stereotomy. On the other hand, Fortifications were still taught using empirical methods based on trial and error until Gaspard Monge solved a problem of *defilement* based on purely geometric work and the use of projections. Problems that had already advanced in their study with Chastillon, but required extensive calculations.

Thus, based on Monge's studies at *the École du Génie de Mézières* and the momentum provided by the French Revolution, Descriptive Geometry emerged in the document creating the *École Centrale des Travaux Publics,* but was initially taught at *the École Normale de l'an III.* His lessons gave rise to the book *Géométrie descriptive: leçons données aux Écoles Normales*, published in 1799. Soon, Architecture and Fortification, based on studies of the projections of three-dimensional objects, came together to form Descriptive Geometry.

An initial study of Monge's lessons revealed that the purpose of these lessons was to conceive Descriptive Geometry as a form of Analysis (in the Enlightenment sense), bringing obviousness, rigor, and generality from the representation of three-dimensional objects in two-dimensional drawings, searching for truth in geometry. Monge sees geometry and algebra as means of achieving mathematical precision and rigor. Initially conceived from an analysis based on Condillac's ideas, the transformation of *the École Centrale des Travaux Publics* into *the École Polytechnique* broke with an educational model inspired by *the École du Génie de Mézières,* in which theoretical training and its applications were integrated. As a result, the *École Polytechnique* became a school of basic education for the other *écoles d'applications.* This concept gradually transformed the teaching of Descriptive Geometry.

Thus, what was observed was an increasing process of understanding the mathematical need to solve engineering problems of solidity, found in Differential and Integral Calculus in the most conducive environment. Thus, the *mathematization* of Architecture took place, causing Descriptive Geometry, in its initial conception, to lose importance in the *École Polytechnique* program *and* give way to Laplace Analysis.

On the other hand, the study identified that the teaching of Fortification was the subject of disputes between the *École Polytechnique* and the application schools, in particular the *École du Génie* in Metz. By claiming all applied teaching of Fortifications for the *École du Génie,* members of *the Corps du Génie*, most of whom had been trained under the *Ancien Régime*, opposed the mathematical processes involved in their art. Thus, the struggle between traditional and modernist engineers led to major crises in teaching, with traditional engineers emerging victorious and returning to Vauban's synthetic and empirical methods in the teaching of Fortifications.

As a result, it was identified that this *mathematization* in Architecture and *demathematization* in Fortifications were the driving forces behind the decline of Descriptive Geometry, since, for the authors of this work, they are the pillars of the discipline of Descriptive Geometry. Thus, even though restricted in its original aspects, descriptive geometry remained a discipline of research and teaching in France until its decline, initially in research and then in teaching.

Conceived from the main forms of Enlightenment philosophy and 18th-century Materialism, Descriptive Geometry underwent changes in its conception as the Enlightenment was replaced by Spiritualism in France, which began during the Napoleonic period. This led to a change in mentality during this period, with an increased appreciation of the humanities and works of literature, and a decline in scientific aspects and science in particular, especially in France. This factor contributed to Descriptive Geometry losing the great social and cultural support that had led to its peak in its origins. Thus, the *École de Monge* had a short life, falling into decline about fifteen years after its creation.

When examining the paths traced in the process of disciplining Descriptive Geometry in Higher Education in France, it was found that there was a departure from Monge's initial conception, making teaching increasingly specialized and distant from practical problems. From this, it can be inferred, especially when this knowledge became a requirement for admission to *the École Polytechnique* and, therefore, a subject of study in secondary education, that Descriptive Geometry has no obligation to vocational training, with its definitive decline in secondary education occurring in the 1960s in France, when it ceased to be taught in secondary schools.

It is worth noting that, throughout a research project, other questions arise and thus fuel the research process. In this sense, the following stand out as a form of continuity for this research:

i. researching the processes that occurred in Descriptive Geometry, which made possible the emergence of new sciences such as Geometric Transformations and Projective Geometry;
ii. identifying the reasons why courses have stopped offering the subject of Descriptive Geometry.

The scope of this article allowed the first author, as an academic and teacher of this subject, to expand on aspects involving the teaching and research of Descriptive Geometry, in addition to a critical reflection on the practice. This article has therefore provided a theoretical deepening of the understanding of all the geometric knowledge that has encompassed the formative and constructive processes since Antiquity, modifying the cognitive structure of the ways of thinking about the teaching of this discipline.

Conflicts of Interest

The author declares that there are no conflicts of interest that could influence the results of the research presented in this article.

Data Availability Declaration

The data collected and analyzed in this article will be made available to the author upon request.

Note

The translation of this paper from Portuguese into English was funded by the Minas Gerais State Research Foundation (Fapemig — Fundação de Amparo à Pesquisa do Estado de Minas Gerais), under Call for Proposals 8/2023.

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