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Between *noesis* and *semiosis*: Analytic Geometry in textbooks through the lens of Semiotic Representations

Abstract: This paper investigates how analytical geometry is addressed in two collections of textbooks approved by the 2021 National Textbooks and Didatic Materials Program. The research is qualitative in nature and supported by the Teorema System. The analysis, based on the Theory of Registers of Semiotic Representation, shows a predominance of treatment and conversion activities, especially calculation and translation between registers. There is a scarce presence of training activities, which suggests a more instructive than exploratory approach. It is concluded that, although the materials mobilize multiple registers, there are limitations in the connections between them, which may restrict the development of broader semiotic skills by students.

Keywords: Semiotics. Textbooks. Duval. Analytic Geometry. Theory of Representations.

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ARTICLE

Entre la noesis y la semiosis: la Geometría Analítica en los libros de texto a través de la lente de las representaciones semióticas

Resumen: Este artículo investiga cómo se aborda la Geometría Analítica en dos colecciones de libros de texto aprobados en el Programa Nacional del Libro y Material Didáctico 2021. La investigación es de carácter cualitativo y está sustentada en el Sistema Teorema. El análisis, basado en la Teoría de Registros de Representación Semiótica, destaca el predominio de las actividades de procesamiento y conversión, especialmente el cálculo y la traducción entre registros. Hay una escasa presencia de actividades formativas, lo que sugiere un enfoque más instructivo que exploratorio. Se concluye que, si bien los materiales movilizan múltiples registros, existen limitaciones en las conexiones entre ellos, lo que puede restringir el desarrollo de habilidades semióticas más amplias por parte de los estudiantes.

Palabras clave: Semiótica. Libros de Texto. Duval. Geometría Analítica. Teoría de las Representaciones.

Entre *noésis* e *semioses*: a Geometria Analítica em livros didáticos sob a lente das Representações Semióticas

Resumo: Este artigo investiga como a Geometria Analítica é abordada em duas coleções de livros didáticos aprovadas no Programa Nacional do Livro e do Material Didático 2021. A pesquisa é de natureza qualitativa, e amparada no Sistema Teorema. A análise, fundamentada na Teoria dos Registros de Representação Semiótica, evidencia predominância das atividades de tratamento e conversão, especialmente cálculo e tradução entre registros. Observa-se escassa presença de atividades de formação, o que sugere uma abordagem mais instrutiva que exploratória. Conclui-se que, embora os materiais mobilizem múltiplos registros, há limitações nas conexões entre eles, o que pode restringir o desenvolvimento de competências semióticas mais amplas pelos estudantes.

Palavras-chave: Semiótica. Livros Didáticos. Duval. Geometria Analítica. Teoria das

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Representações.

1 Introduction

The research presented in this paper is part of the research that the TeorEMa — Interlocutions between Geometry and Mathematics Education group has been developing on the theme of the presence of Geometry in Mathematics textbooks (TB), especially with regard to changes in the New High School (NHS) program.

In Brazil, it is common practice to use textbooks as a support for teachers in their classes, who are free to integrate them with other media, such as the internet, videos, and concrete materials, as highlighted by Guimarães et al. (2007). Borba, Azevedo, and Bittar (2016, p. 1) observe that "textbooks can indicate which concepts are selected for teaching and how they should be worked on in school".

The results shared in this text refer to a study focusing on analytical geometry, whose objective was to understand how it is present in a sample of NHS textbooks. For this analysis, the Theory of Registers of Semiotic Representation (RSR), proposed by Duval (1998), was fundamental. Other studies have also analyzed textbooks in light of the RSR, such as Santana, Gualandi, and Soares (2019) and Silva, Prando, and Gualandi (2020), which reinforces its methodological relevance. But how can we identify the presence of Analytical Geometry, since, as Machado (2021) stated, "no mathematical object has materiality"?

This statement leads to reflection and some questions, such as: Without materiality, how can mathematical objects be accessed/understood? Machado (2021) states that access occurs through representations. He further observes that, in order to lead a student to construct a mathematical object, "we must use as many representations of that object as possible and convert one representation into another several times. The conversions make it possible to identify that they are representations of the same object" (Machado, 2021, p. 5).

And why would we have different representations of the same object? Machado (2021, p. 6) clarifies that each representation of a mathematical object "highlights a characteristic of that object, each representation sheds light on an aspect of the object, and the perception that all representations, with their highlights, refer to the same object allows for the mental construction of the object".

For Duval (1998), each representation system — natural language, symbolic language, diagrams, geometric figures, Cartesian graphs, tables, among others — presents different learning issues. Machado (2021, p. 8) observes that the "choice of register is usually linked to the simplicity and economy of treatment" of the topic under study.

These reflections underpin this text, which is in line with the theme of this edition. It is assumed that the way content is presented has a direct impact on teaching practices and student learning.

By analyzing how Analytical Geometry is treated in textbooks, potential gaps or strengths in the approach to content are identified, providing input for the development of materials that better meet the needs of teachers and students. Thus, the research aims to contribute to the improvement of curricula, fostering a link between theory and practice and promoting the development of students' cognitive and mathematical skills.

2 Theory Registers of Semiotic Representation

Raymond Duval, a French philosopher and psychologist, wrote the book *Sémiosis et pensée humaine: registres sémiotiques et apprentissages intellectuels* (Duval, 1995), which was translated into Portuguese as *Semiósis e pensamento humano: registros semióticos e* *aprendizagens intelectuais* (Duval, 2023). In this work, the author developed a theory that makes it possible to understand how students construct mathematical knowledge and develop cognitive skills linked to RSR.

To this end, it is essential to understand that mathematical symbols, also known as signs, play a crucial role in mathematical language. Just as it is necessary to distinguish the object from its representations (Duval, 1998), signs allow the expression of numerical concepts and relationships, thus serving as a form of representation. It is essential to recognize that each mathematical sign represents a specific concept or operation, that is, it represents an idea, and that they can be combined in various ways to form expressions and equations that describe complex mathematical phenomena, thus representing objects.

According to Duval (1998), the Theory of RSR involves two main ideas: *semiotics* and *mental*. *Semiotic* representations are a simple means of externalizing *mental* representations. These representations are intrinsically linked to semiosis, that is, the production and interpretation of signs (*Semiósis*). Directly linked to it is *Noesis*, which concerns the conceptual apprehension of a mathematical object. Thus, it is understood that while *Semiósis* addresses the semiotic representation of signs, *Noésis* addresses their mental representations. Therefore, there is no *Noésis* without *Semiósis* — that is, there is no way to understand a conceptual object without it being represented by signs.

Based on the author's ideas, some of the types of registers present in textbook were identified: the mother tongue, which represents the object in natural language, such as its writing in Portuguese; the algebraic register, which consists of the direct algebraic representation of the object; the graphic register, which uses visual elements for representation; the numerical register, which represents the object in numerical form, among others, outlined in Figure 1

Figure 1: Types of records diagram (Own elaboration, 2025)

According to Machado (2021), different registers enable distinct ways of expressing mathematical concepts, each with its own advantages/limitations and specific applications. The objects of knowledge, including mathematical entities, can only be accessed through their representations. Thus, each representation of an object sheds light on a particular aspect of the object, and conceptual understanding (*Noésis*) is established by the perception that all these representations refer to the same mathematical object.

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3 Cognitive activities linked to semiosis

Considering the types of RSR mentioned above and their direct relationship with semiosis, Duval (1998) presents three types of cognitive activities: *formation*, *treatment*, and *conversion*. The first, *formation,* refers to knowledge of the rules of conformity; the second, *treatment*, addresses an internal transformation of the register; in other words, *treatment* can be defined as a transformation of one representation into another in the same register; and the third, *conversion*, occurs through external transformations of the register, that is, between different registers (Figure 2).

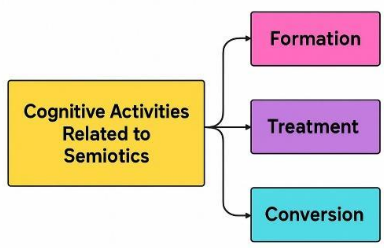

Figure 2: Diagram of cognitive activities linked to semiosis (Own elaboration, 2025)

It is important to note that both treatment and conversion can be subdivided into several distinct forms, as shown in Figure 3. Treatment is subdivided into three forms: calculation, which is the treatment itself in algebraic notation; paraphrase, which is in natural language; and anamorphosis, which refers to the treatment of all figural representations. Conversion is also subdivided into three other forms: illustration, which occurs based on the conversion of a linguistic representation into a figural representation; translation, which addresses a linguistic representation in a given language to an algebraic or numerical representation; and description, which starts from a nonverbal representation (schema, figure, and graph) to a linguistic representation.

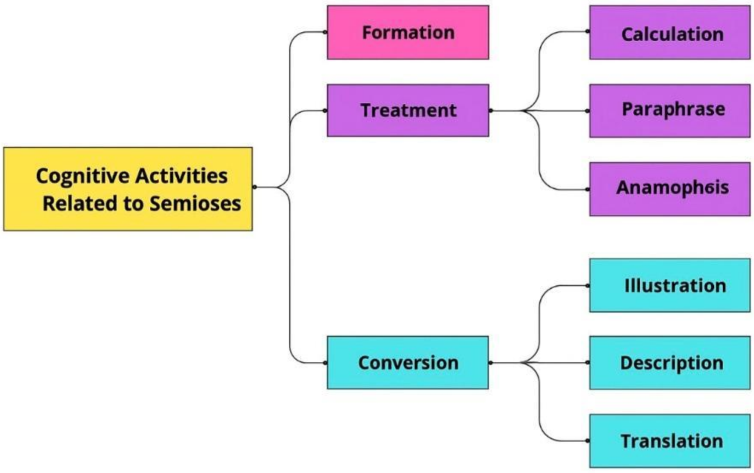

Figure 3: Diagram of cognitive activities linked to semiosis and their types (Own elaboration, 2025)

4 Cognitive activities close to conversion

In addition to cognitive activities linked to semiotics, there are cognitive activities close to conversion (Figure 4). In this context, it is important to distinguish between transformations and conversion, because although they can be confused, they refer to different processes. Transformations include *encoding*, which is the *transcription* of a representation into a semiotic system different from the one initially given; and *interpretation*, which requires a change of theoretical framework or a change of context. This change does not imply a change of register.

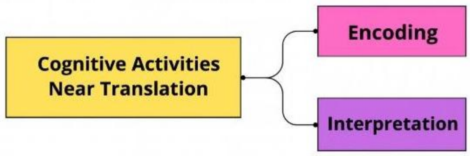

Figure 4: Diagram of cognitive activities close to conversion (Own elaboration, 2025)

5 Methodology

The research methodology used in this study is based on a qualitative approach. As Bogdan and Biklen (1994, p. 209) state, studies of this nature "should reveal greater concern for the process and meaning rather than for its causes and effects." In this context, the focus is on the presence of Analytical Geometry in NHS Mathematics TB, analyzing the different approaches to geometric content, rather than on results or products. This reflects one of the central characteristics of qualitative research, as pointed out by Denzin and Lincoln (2000) and Vidich and Lyman (2000).

Thus, this approach is considered appropriate for investigating how geometric concepts are presented and provided opportunities in the NHS Mathematics TB, particularly in the context of Analytical Geometry, considering the RSR proposed by Duval (1998) as theoretical support.

The methodological approach was based on the Teorema System (Amaral *et al.*, 2022), summarized in Figure 5.

The selection of the books *Conexões: Matemática e suas Tecnologias: Matrizes e Geometria Analítica* [Connections: Mathematics and its Technologies: Matrices and Analytical Geometry] (Leonardo, 2020) and *Quadrante: Matemática e suas Tecnologias:Sistemas Lineares e Geometria Analítica* [Quadrant: Mathematics and its Technologies: Linear Systems and Analytical Geometry] (Chavante and Prestes, 2020) for the analysis of GA content, among the ten collections approved in *Programa Nacional do Livro e do Material Didático* [National Textbooks and Didatic Materials Program — PNLD], 2021 edition, (Brasil, *website*[1]), is based on the fact that only four explicitly mentioned Analytic Geometry in the title, suggesting a structured organization of this content. Initially, the analysis was directed at the textbooks in these four collections; however, due to time constraints, it was only possible to examine two, selected at random.

Based on the Teorema System and assuming the textbook as an independent object of study, the exploration of the material was guided both by its structural characteristics and by the specific aspects of this investigation. This approach allowed us to understand that the data would be processed through *analysis*. The tasks[2] proposed in the books were analyzed, examining them according to the RSR Theory, and throughout this process, the tasks were organized and filed.

That said, some criteria were established based on the RSR, which include identifying the variety of representation registers used — such as graphical, algebraic, and numerical — and the clarity in the transition between these registers. Considering that the RSR Theory is not intuitive, conversions, types of interaction in the register itself (treatment), Cognitive Activities Close to Conversion, and, finally, types of treatment and conversion were observed.

[1] Access link: https://pnld.nees.ufal.br/pnld_2021_didatico/componente-curricular/pnld-2021-obj2-matematica-e-suas-tecnologias. Accessed on: April 22, 2025.
[2] The term *task* is used as defined in Litoldo (2021).

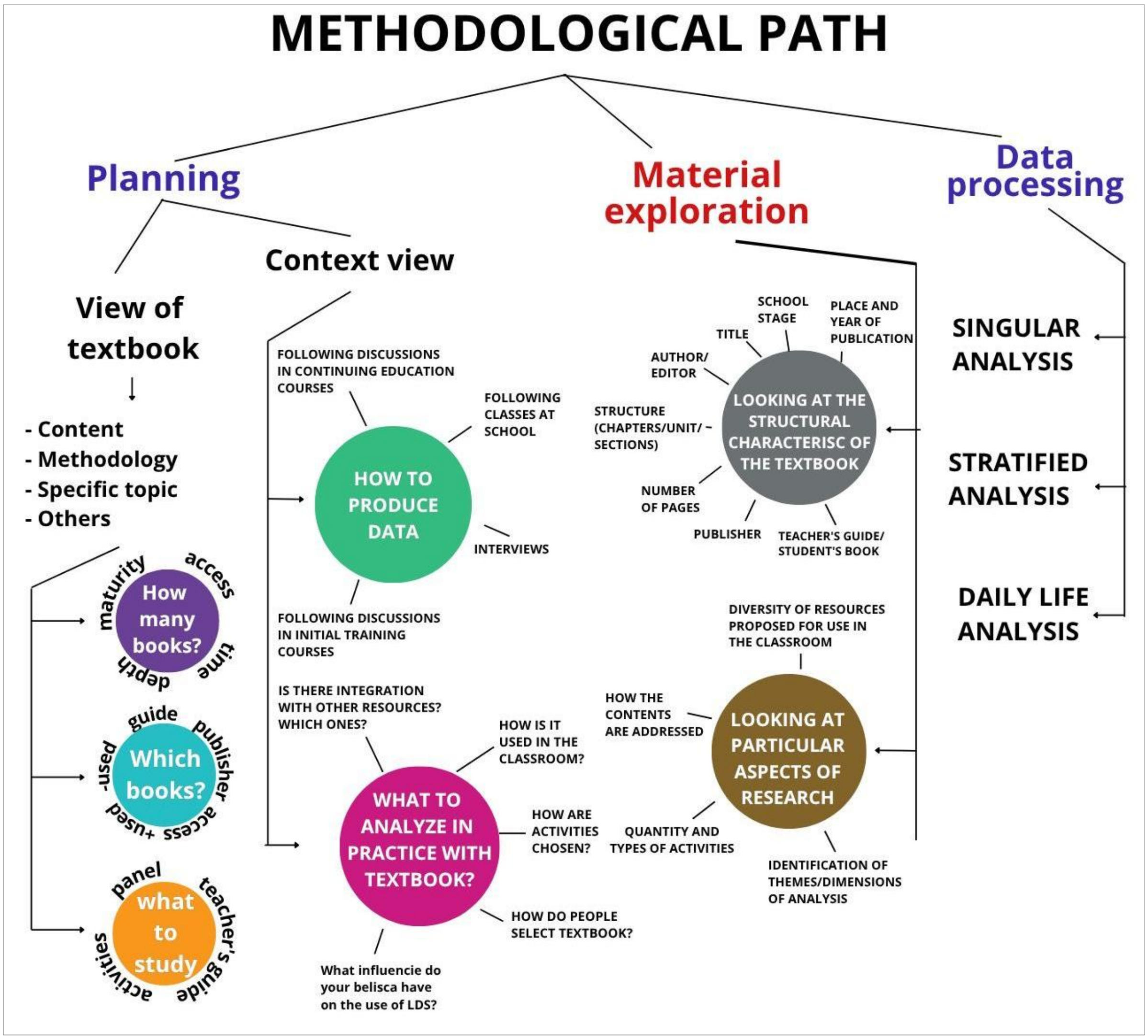

Figure 5: Teorema System (Amaral *et al*., 2022, p. 202)

6 Results

This section presents examples of tasks related to each of the Cognitive Activities Linked to Semiosis and their respective dimensions, together with discussions based on the RSR Theory (Duval, 1998). Next, we discuss the classification of the two books analyzed, based on quantitative data expressed in graphs, with the aim of reflecting on the content proposed in these materials. The objective is not to compare them, but to gain a broader understanding of what has been offered in terms of cognitive activities linked to semiosis and how these activities have been distributed.

6.1 Building the scenario

The analyses focus on the different types of representation registers and the interactions between them. Next, examples are presented that illustrate these cognitive activities linked to semiosis in the context of Analytic Geometry, highlighting how transformations occur through semiotic registers. Next, cognitive activities close to conversion are presented. These examples provide a more concrete understanding of the application of the concepts of treatment and conversion, illustrating how the tasks present in TB address the complexity of transitions between registers and their importance for student learning.

Cognitive Activities Linked to Semiosis

1. Treatment

In Analytical Geometry, the study of transformations between different RSR is essential for the development of mathematical thinking. In this context, treatment activities represent internal manipulations, in which students work in the same register, applying algebraic operations and manipulations without the need to move to another representation. These activities are fundamental in TB, as they allow students to deepen their understanding and mastery of concepts without switching between different registers, which can promote more solid learning of a given concept.

In this section, examples of treatment and conversion in analytical geometry tasks found in the TB analyzed are explored. The examples will serve to illustrate how these operations are approached.

1.1 Calculation

Among the cognitive activities related to semiosis, the types of treatment mentioned above in Figure 3 are exemplified. Figure 6 shows a task that asks students to obtain the values of variables *m* and *n*, requiring an understanding of the concept of the *second quadrant* and its properties. Figure 7 shows the authors' solution (from the Teacher's Manual) and, in this example, the calculation is evident during the solution of the problem, since it is an algebraic process that does not involve switching between different types of registers. Cognitive skills related to semiosis are exemplified by the types of *treatment* mentioned above in Figure 3. Figure 6 presents a task that requires students to obtain the values of the variables m and n, which requires an understanding of the concept of the second quadrant and its properties. Figure 7 shows the authors' solution (from the Teacher's Manual) and, in this example, the calculation is evident during the resolution of the problem, since it is an algebraic process that does not involve switching between different types of records.

4- For the values of m and n does the point A(m - 8, n - 5) belong to the 2nd quadrant? Explain how you obtained these values.

Figure 6: Example of a task focusing on calculation (Leonardo, 2020, p. 64)

Treatment
Calculation

The points belonging to the 2nd quadrant have coordinates x ≤ 0 and y ≥ 0. So, to obtain the values of m, we must do m - 8 ≤ 0 and, to obtain the values of n, we must do n - 5 ≥ 0.

Thus: m - 8 ≤ 0 → m ≤ 8
n - 5 ≥ 0 → n≥ 5
Therefore, m and n E R such that m ≤ 8 and n ≥ 5.

Figure 7: Task resolution (Leonardo, 2020, p. LXXVI, notes from the authors)

Thus, its central role in the development of mathematical reasoning in the context of GA is emphasized. Calculation, as an algebraic activity, allows students to manipulate expressions and solve problems internally in the algebraic register, developing fundamental skills for understanding geometric relationships and properties. However, it is crucial that this practice is not limited to mechanical operations.

The lack of integration with other registers, such as graphs or the mother tongue, can lead to a fragmented understanding of concepts, in which students know *how to do* something without understanding *why*. Thus, calculation should be promoted not only as a procedure, but as a means of constructing mathematical meaning, allowing students to articulate the algebraic with the visual and verbal to achieve a richer and more integrated understanding of GA.

1.2 Paraphrase

Paraphrasing is an internal transformation of the register that acts on the rewriting of texts in the mother tongue, aiming to express the same idea of a statement through different words and structures, while maintaining the original meaning. In the context of the tasks set out in the GA content, paraphrasing can be seen as a comprehension tool, facilitating understanding of the problem proposed and its resolution.

Figure 8, for example, presents a case in which the task statement is rephrased in clearer language, helping to interpret the problem and, consequently, to execute the solution more efficiently. This linguistic transformation, by reworking the original content, constitutes a paraphrase.

A rural cell phone antenna covers a circular region with an area equal to 900 π km^2. This antenna is located in the center of the circular region and its position in the Cartesian system, with measurements in kilometers, is the point (0,10).
Thus, the equation of the circumference that delimits the circular region is: **(a)**

a) x + y - 20y - 800 = 0.
b) x + y - 20x - 70 = 0.
c) x + y - 20x - 800 = 0.
c) x + y - 20y + 70= 0.
e) x + y = 900.

The area A of a circle of radius r is given by: A= πr^2

Treatment
Paraphrase

Since the area of the circular region covered by this antenna is 900π km^2, we have:

$900\pi = \pi r^2 \rightarrow r^2 = 900 \rightarrow r = 30$

Therefore, we have a circle with center C(0,10) and radius 30, whose equation is:
$(x-0)^2 + (y-10)^2 = 30^2 \rightarrow x^2 + y^2 - 20y + 100 = 900 \rightarrow x^2 + y^2 \rightarrow 20y - 800 = 0$
Therefore, with the obtained answer, we conclude that alternatives **b, c, d** and **e** are incorrect. Therefore, the correct alternative is **a**.

Figure 8: Example of task and resolution with paraphrasing (Own elaboration based on Chavante and Prestes, 2020, p. 128 and 106, respectively)

In summary, by adopting paraphrasing at the beginning of task resolutions, as illustrated in Figure 8, we seek to improve communication of the problem and assist students in understanding the steps involved. The practice of paraphrasing thus contributes to clarity and effectiveness in the presentation of solutions, without losing fidelity to the original content.

1.3 Anamorfoses

Figure 9 presents two tasks, 49(a) and 49(b), that exemplify anamorphosis by asking students to add vectors. In this context, the grid on which the vectors are arranged can be interpreted as a figural register. The transformation involved in the sum of vectors in this scenario is not only an algebraic process, but also a transformation of the figural representation, since the vectors are visually displaced and adjusted on the grid to find the sum vector.

According to Duval (1998), anamorphosis is defined as a transformation that alters the spatial arrangement of a figural representation without modifying the internal characteristics of the objects represented. In this example, the arrangement of the vectors in the grid is adjusted to reflect the vector sum, preserving their geometric and relational properties, but repositioning them to highlight the resulting vector.

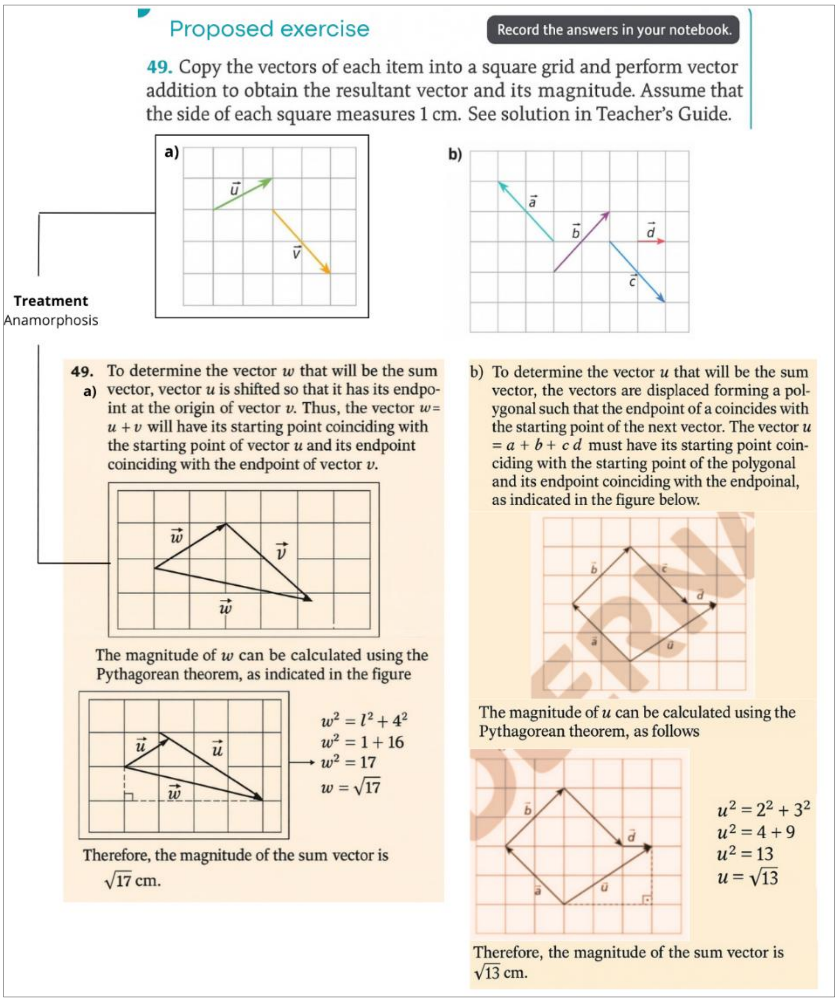

Figure 9: Example of task and solution highlighting anamorphosis (Own elaboration based on Leonardo, 2020, p. 86 and LXXV, respectively)

Thus, it can be said that these tasks incorporate the concept of anamorphosis by exploring transition and manipulation in the figural register in a heuristic and visual way, without altering the algebraic calculations, which remain in the symbolic register.

2. Conversion

The examples of treatment analyses show how students are encouraged to work with different transformations in the same register, deepening their mastery of mathematical rules and properties. However, understanding mathematical concepts requires students not only to operate in a single register, but also to make transitions between different representations.

As mentioned earlier, this process, called conversion, is fundamental to more comprehensive learning and will be explored below in its different forms — illustration, translation, and description — with examples that illustrate how TB address this complex and essential cognitive skill.

2.1 Illustration

Starting with the conversion examples in Figure 10, task 5(a) asks students to identify and represent certain points on the Cartesian plane. The task involves a conversion between registers, since the student starts from the vector language of points P(8,0) and Q(0,6) and represents them graphically on a Cartesian plane. This conversion process is characterized by the transition from vector representation to graphical representation (figural register), in which the coordinates are visualized on the plane.

5- a) Solve the folloing itens
Construct a Cartesian plane and represent the points on it P(8,0) and Q(0,6)

Figure 10: Example of a task with illustration (Leonardo, 2020, p. 64)

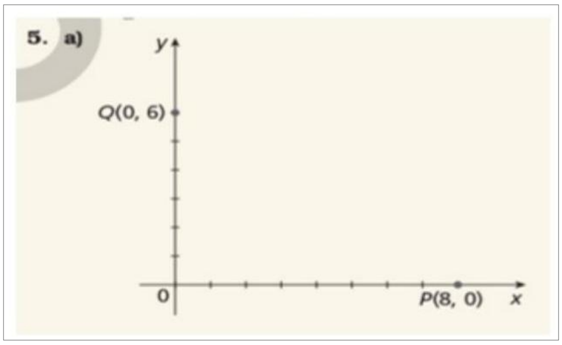

Figure 11: Task resolution (Leonardo, 2020, p. LXXVI)

In this way, the illustration highlights its pedagogical value in teaching analytical geometry. The conversion from vector representation to graphical representation allows students to visualize and interpret mathematical concepts, strengthening their understanding of geometric relationships.

The illustration process serves as a bridge between the abstract and the concrete, facilitating learning by transforming coordinates and vectors into tangible positions on the Cartesian plane. However, the effectiveness of this conversion depends on understanding the meaning of each coordinate and its relative positions. Thus, illustration aids in the assimilation of mathematical concepts, functioning as a powerful tool that unites visualization and interpretation.

2.2 Description

To illustrate conversion as a *description*, task 3(a) is presented (Figure 12). In this task, the opposite process to the previous example occurs: instead of plotting points on the graph, students are asked to identify the coordinates of the vertices of a polygon already drawn on the Cartesian plane. This process suggests the conversion of graphic language (polygon on the plane) to vector language (coordinates of points).

According to the theoretical contribution of RSR, this conversion corresponds to a transition between two distinct semiotic registers: the student starts from a nonverbal representation (the drawing of the polygon in the graph) and transforms it into a vector representation (the numerical coordinates of the vertices), thus characterizing a description.

In this process, the student must visually interpret the positions of the vertices and express them symbolically, which requires both the precise identification of points on the plane and mastery of vector language. This mechanism is particularly interesting because it replicates the conversion performed in the illustration, allowing the student to travel back and forth between the graphic and vector registers.

This ability to move between the two senses strengthens the student's understanding and autonomy, as it enables the visual interpretation of the positions of the vertices and their symbolic expression.

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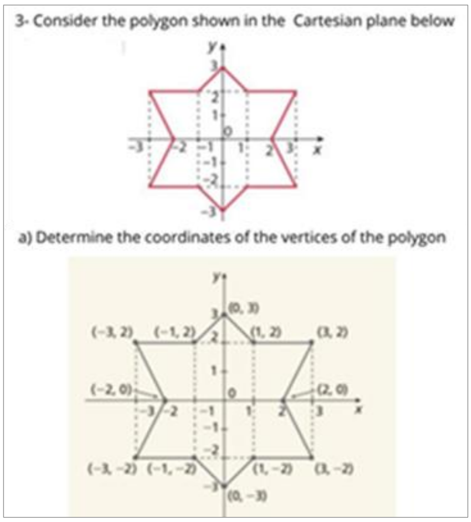


Figure 12: Example of task and resolution in description conversion (Own elaboration based on Leonardo, 2020, p. 63 and LXXVI, respectively)

2.3 Translation

An example that involves can be seen in the task in Figure 13. Here, the student is asked to construct a Cartesian plane and locate certain points based on the coordinates provided. It can be seen that this task mobilizes the student's knowledge in three types of language, highlighting vector language (point coordinates) and algebraic language (translation of coordinates into numerical expressions). This task explores the process of conversion between different semiotic registers. The student initially performs a conversion between the vector register (points described by coordinates) and the algebraic register, identifying the numerical coordinates.

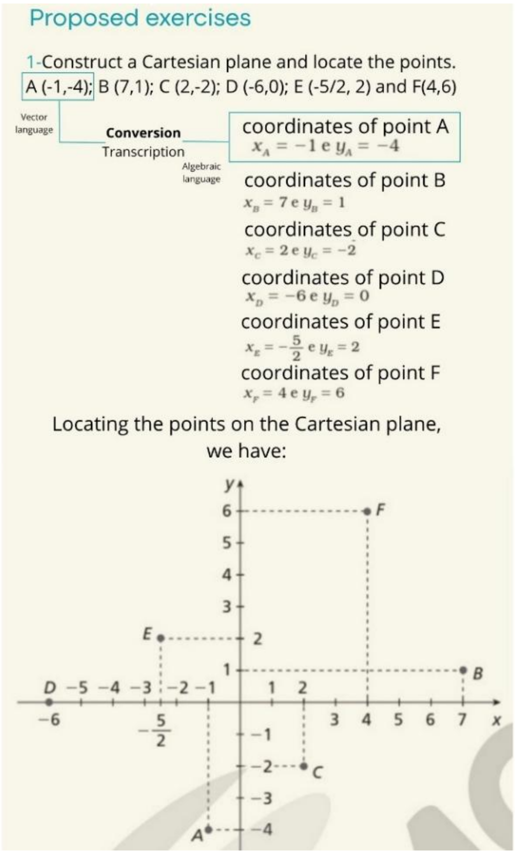


Figure 13: Example of task and solution with translation. (Own elaboration based on Leonardo, 2020, p. 63 and LXXVI, respectively)

The nature of the conversions present in the task in Figure 13 reflects a process of translation between different forms of representation, which is relevant for a broader and deeper understanding of the mathematical concepts involved.

According to Duval's theory (1998), the ability to perform these conversions is essential for learning, as it enables students to understand how different representations relate to each other and contribute to the construction of mathematical meanings. This type of task promotes the development of students' semiotic competence by requiring them to navigate different registers and translate information between them.

Cognitive Activities Close to Conversion

The examples discussed so far highlight the diversity of cognitive activities linked to semiosis that TB offer, both in *treatment* and conversion. These activities allow students not only to understand mathematical concepts in isolation, but also to develop the ability to operate between different registers of representation — an essential aspect for a deeper and more flexible understanding of mathematics, as proposed by Duval (1998).

Although these examples illustrate how the processes of treatment and conversion are presented, there are other cognitive activities mentioned above that, although they do not directly involve conversion, require specific skills of adaptation between registers. These are called conversion-related cognitive activities (Figure 4), which also play a fundamental role in the construction of mathematical knowledge.

To begin the analysis of these activities, we will address coding and interpretation. Although they do not involve a direct conversion between different registers, these practices are fundamental to mathematical understanding, since they require adaptations and transitions within the same register or between close registers. To illustrate this, Figures 14 and 15 present a task that requires the application of the *Pythagorean Theorem*. Transcribing the idea of this theorem into its respective formula constitutes a coding.

7- Calculate the distance between the points of each item.

a) $A(2, 1)$ e $B(5, 5)$ 5 **c)** $D(-4, -2)$ e $E(0, 7)$ $\sqrt{97}$

b) $A(0, 0)$ e $B(-1, 3)$ $\sqrt{10}$ **d)** $C(4\sqrt{3}, 5)$ e $B(6\sqrt{3}, 3)$ 4

Figure 14: Task of exemplifying the description (Leonardo, 2020, p. 66)

7. a) Temos: $A(2, 1)$ e $B(5, 5)$

$$d_{A,B} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

$$d_{A,B} = \sqrt{(5-2)^2 + (5-1)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

b) Temos: $A(0, 0)$ e $B(-1, 3)$

$$d_{A,B} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

$$d_{A,B} = \sqrt{(-1-0)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$$

c) Temos: $D(-4, -2)$ e $E(0, 7)$

$$d_{D,E} = \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2}$$

$$d_{D,E} = \sqrt{(0+4)^2 + (7+2)^2} = \sqrt{16+81} = \sqrt{97}$$

d) Temos: $C(4\sqrt{3}, 5)$ e $B(6\sqrt{3}, 3)$

$$d_{C,B} = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2}$$

$$d_{C,B} = \sqrt{\left(6\sqrt{3} - 4\sqrt{3}\right)^2 + (3-5)^2} = \sqrt{12+4} = 4$$

Figure 15: Resolution from the Teacher's Book illustrating the description (Leonardo, 2020, p. LXXVII)

In this context, the student makes a transition between verbal language and algebraic representation, converting the problem statement into a mathematical expression that can be manipulated algebraically. This coding plays an essential role in allowing the student to connect the conceptual meaning of the theorem to its practical and operational representation, facilitating both understanding and application in geometric problems. It should be noted that, even if the resolution does not explicitly mention the name of the theorem or the answer directly, these elements are implied in the development of the solution provided in the textbook (Teacher's Manual).

Coding is not a conversion, because although both refer to the same concept, they do not express the same idea in an identical way. In coding, the student translates the verbal concept of the *Pythagorean Theorem* into its formula, without describing the meaning of the theorem in detail in words. Duval (1998) defines this transcription as coding, since there is no complete change of register, but rather a transposition that preserves the same conceptual meaning, allowing the student to manipulate and apply the concept in the form of an algebraic expression.

Based on the same example, interpretation involves a change of theoretical framework or context. This can be seen in the conclusions of the tasks, when, for example, in letter *a*, it is stated that the distance between two points is 5. In this case, meaning is being attributed to the number 5, interpreting it in the context of the problem, ceasing to be just an abstract value to become the concrete measure of the distance between two specific points on the plane.

Thus, interpretation requires students to assign contextual meaning to the result obtained, connecting it to the problem situation and understanding its practical application. This process is fundamental, as it transforms numerical values and mathematical results into information relevant to understanding and solving the proposed problem.

6.2 Discussing the scenario

Based on the data produced in both collections, these were initially arranged in tabular form (Figure 16 and 17). Each cognitive activity was quantified in terms of absolute frequency (n) and percentage (%), enabling a comparative analysis between the types of activities proposed in the two books. Subsequently, graphs were generated to summarize the percentage distribution of the RSR. The quantifications generated show a similarity in the results of the two collections analyzed.

			Activities			
			Tasks			
Author — TB			**n**	**%**	**N**	**%**
Leonardo (2020)	**Cognitive Activities Linked to Semiosis**					
	Formation		0	0	0	0
	Treatment	**Calculation**	309	46,18834	303	45,29148
		Paraphrase			0	0
		Anamorphosis			6	0,896861
	Conversion	**Illustration**	360	53,81166	36	5,381166
		Description			36	5,381166
		Translation			288	43,04933
	Not Applicable		0	0	0	0
	Total		669	100	669	100
	Cognitive Activities Near Conversion					
	Coding		320	55,36332		
	Interpretation		258	44,63668		
	Not Applicable		0	0		
	Total		578	100		

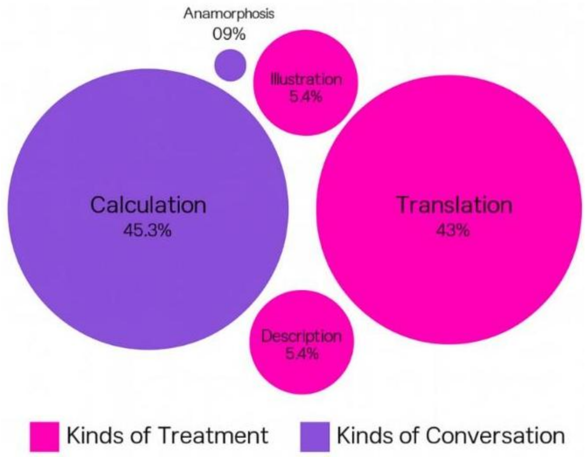
Figure 16: Tabulation of data in *Connections: Mathematics and its Technologies: Matrices and Analytical Geometry* (Own elaboration, 2025)

			Activities			
			Tasks			
Author — TB			**n**	**%**	**N**	**%**
Chavante (2020)	**Cognitive Activities Linked to Semiosis**					
	Formation		6	0,771208	6	0,771208
	Treatment	**Calculation**	374	48,07198	364	46,78663
		Paraphrase			9	1,156812
		Anamorphosis			1	0,128535
	Conversion	**Illustration**	398	51,15681	38	4,884319
		Description			23	2,956298
		Translation			337	43,3162
	Not Applicable		0	0	0	0
	Total		778	100	778	100
	Cognitive Activities Near Conversion					
	Coding		232	53,08924		
	Interpretation		205	46,91076		
	Not Applicable		0	0		
	Total		437	100		

Figure 17: Tabulation of data from *Quadrant: Mathematics and its Technologies: Linear Systems and Analytical Geometry* (Own elaboration, 2025)

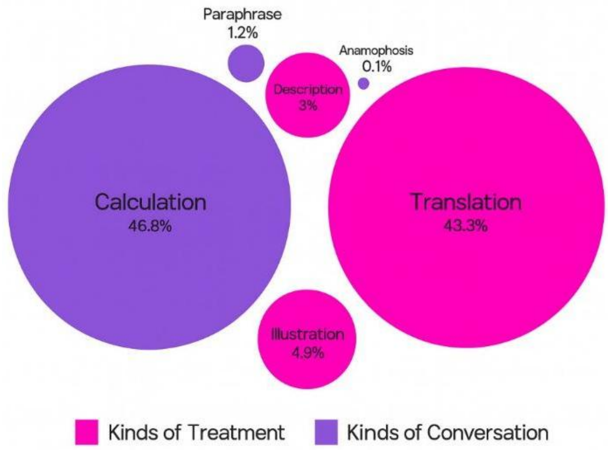
In Leonardo's collection (2020), cognitive activities linked to semiosis are divided evenly between treatment (46.2%) and conversion (53.8%). There were no formative activities in this collection, and the classification of cognitive activities close to conversion was more uneven, representing 55.4% of coding and 44.6% of interpretation.

In tasks involving treatment, there is a disparity between their forms, with calculation being predominant (45.3%) in relation to anamorphosis (0.9%), in addition to the absence of paraphrasing in task resolutions. In conversion, translation stands out as the most common type (43%), followed by illustration (5.4%) and description (5.4%) in equal percentages.



Graph 1: Cognitive activities linked to semiosis in Leonardo (2020)

In the collection by Chavante and Prestes (2020), the distribution of cognitive activities linked to semiosis is similar, with treatment (48%) and conversion (51.2%) appearing in comparable proportions. In this case, formation appears very little, with only 0.8% occurrence, while the classification of Cognitive Activities Close to Conversion represents 61.2% of coding and 38.8% of interpretation. Among the forms of treatment, although all are present, calculation appears as the main type (46.8%), significantly surpassing paraphrase (1.2%) and anamorphosis (0.1%), whose percentages are minimal. In conversion, translation also stands out (43.3%), followed by illustration (4.9%) and description (3%).



Graph 2: Cognitive activities linked to semiosis in Chavante and Prestes (2020)

When analyzing cognitive activities close to conversion, both coding and interpretation have a considerable presence in both books. In Leonardo (2020), coding accounts for 55.36% of activities close to conversion, while interpretation represents 44.64%. In Chavante and Prestes (2020), the distribution is similar, with coding at 61.17% and interpretation at 38.83%.

These data reflect the relevance attributed to the ability to represent mathematical concepts and interpret these symbols in different contexts, which are essential skills for the development of mathematical thinking.

Looking at the two collections from a broader perspective, it is clear that training activities occur infrequently, possibly because the analysis is focused on tasks. When it does appear, it is usually intended for teachers, as part of the solutions proposed in the Teacher's Manual, which suggests a possible limitation in promoting activities that encourage students to create their own representations.

Training would allow students not only to use records but also to develop their own representations, stimulating a higher level of reflection and understanding of concepts. The lack of emphasis on this aspect may indicate a more instructional and less exploratory approach, focused on the application of predefined concepts rather than fostering the autonomous construction of new meanings.

The predominance of *treatment* and conversion activities is consistent with the role of these processes in Duval's RSR Theory, which highlights the importance of flexibility between registers for a deep understanding of mathematics. However, the greater emphasis on internal *treatment* activities, such as calculation, compared to activities that encourage creation, such as formation, and freer interpretation suggests that both books follow a more traditional and controlled teaching structure.

The traditional approach has its advantages, especially for the development of operational skills, but an expansion to include more formation and interpretation activities could help students develop a more critical and creative understanding of mathematics. Increasing these activities would promote students' ability to not only follow mathematical processes, but also to question, adapt, and expand mathematical concepts to new contexts.

As for the types of treatment, calculation stands out significantly, while paraphrasing is not even recorded in one of the collections. In conversion, translation stands out, as it is widely used in tasks involving, for example, the application of formulas, which is used with significant frequency in almost all tasks. With regard to cognitive activities close to conversion, it is noted that coding and interpretation occur frequently, appearing in almost all tasks, in many cases more than once.

7 Final considerations

Based on the data presented and the discussions conducted, it can be concluded that the TB analyzed offer a variety of RSR, but with a notable concentration in certain cognitive activities, especially treatment through calculation and conversion predominantly via translation.

This distribution suggests a tendency for the materials to favor the direct operationalization of mathematical concepts, which, as argued by Duval (1998), only partially meets the needs for a full and flexible understanding of concepts.

Despite the presence of multiple registers in the TB, explicit connections between them are limited, which may restrict students' ability to perform conversions between different forms of representation — an essential skill for the construction of broad and deep mathematical thinking. The scarcity of tasks that promote more complex transitions between registers, such as paraphrasing and anamorphosis, suggests an opportunity to broaden the approach to content, especially in topics such as analytical geometry, which would benefit from a diversity of representations and interactions.

These analyses indicate the importance of enriching teaching materials with tasks that promote not only the presentation of multiple representations, but also the understanding and exploration of the relationships between them. Strengthening these connections can enhance the development of students' semiotic skills, enabling more comprehensive and potential mathematical learning.

Thus, it is worth considering that future revisions and elaborations of teaching materials take into account this diversity of approaches, aiming at the formation of students capable of moving with flexibility and depth between different mathematical registers.

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Conflicts of Interest

The authors declare no conflicts of interest that could influence the results of the research presented in the article.

Data Availability Statement

The data collected and analyzed in the article will be made available upon request to the authors.

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