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Educação Matemática Debate

An analysis of the context of reality in Geometry tasks in New High School textbooks

Abstract: What are the different contextual references present in Geometry tasks in New High School (NEM) textbooks? This question guides the qualitative research, which adopts the Theorem System as its methodological approach, based on contextual references. A total of 7,345 Geometry tasks from NEM textbooks were classified. This paper presents an analysis focused on Reality tasks. The results show that most contexts are purely mathematical, while a minority refer to reality. In addition, some of these contexts mention aspects of reality in their statements, but the question proposed is situated in other contexts, such as Pure Mathematics, losing part of its power to connect everyday life and the concepts studied.

Keywords: Contextualization. Textbook. New High School. Reality in Tasks. PNLD.

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ARTICLE

Un análisis del contexto de las tareas de Realidad en Geometría en los nuevos libros de texto de Nuevo Bachillerato

Resumen: ¿Cuáles son las distintas referencias de contexto presentes en las tareas de Geometría en los Libros de Texto (LT) del Nuevo Bachillerato (NB)? Esta pregunta orienta la investigación, de carácter cualitativo, que adopta como enfoque metodológico el Sistema Teorema, basado en las referencias de contexto. Se clasificaron 7.345 tareas de Geometría de los LT del NB. En este artículo se presenta un análisis centrado en las tareas con contexto de Realidad. Los resultados muestran que la mayoría de los contextos son Puramente Matemáticos, mientras que solo una minoría hace referencia a la Realidad. Además, parte de estos contextos menciona aspectos de la realidad en sus enunciados, pero la cuestión propuesta se sitúa en otros contextos, como el de la Matemática Pura, perdiendo parte de su potencial para conectar la vida cotidiana con los conceptos estudiados.

Palabras clave: Contextualización. Libro de Texto. Nuevo Bachillerato. Contextos Reales. PNLD.

Uma análise do contexto de Realidade nas tarefas de Geometria dos livros didáticos do Novo Ensino Médio

Resumo: Quais são as distintas referências de contexto presentificadas nas tarefas de Geometria nos Livros Didáticos (LD) do Novo Ensino Médio (NEM)? Essa questão norteia a pesquisa, qualitativa, adotando como percurso metodológico o Sistema Teorema, com base nas referências de contexto. Foram classificadas 7.345 tarefas de Geometria dos LD do NEM. Neste artigo, apresenta-se uma análise focada nas tarefas de Realidade. Os resultados mostram que a maior parte dos contextos é Puramente Matemático, enquanto uma minoria se referiu à Realidade. Além disso, parte desses contextos menciona aspectos da realidade em seus enunciados, mas a questão proposta se situa em outros contextos, como da Matemática Pura, perdendo parte de sua potência de conexão entre o cotidiano e os conceitos estudados.

Palavras-chave: Contextualização. Livro Didático. Novo Ensino Médio. Realidade em Tarefas.

PNLD.

1 Introduction

With the New High School (NEM), many concerns have arisen in the educational field, especially regarding the dynamics of this change in schools, particularly in teaching materials. These issues have been the focus of interest of the research group teorEMa — Interlocutions between Geometry and Mathematics Education.

Textbooks are one of the main resources used in the classroom (Amaral *et al.*, 2022). Since the approval of the Base Nacional Comum Curricular [National Common Curriculum Base — BNCC (Brasil, 2017), textbooks have been produced in such a way as to become the material that effectively implements the BNCC, since they follow its regulations and are supported by the Programa Nacional do Livro e do Material Didático [National Program for Textbooks and Teaching Materials — PNLD.

In this context, this study aims to share the results of a research project whose problem can be summarized as follows: what are the different contextual references present in Geometry tasks in Mathematics textbooks for the new high school curriculum? We seek a possible contribution from teachers and researchers in the field of Mathematics Education and, albeit indirectly, from the PNLD, since discussions arising from research such as this can influence both the authors of these materials and the PNLD evaluators and teachers involved in the selection process.

When dealing with different context references, the analysis is based on the theoretical contribution discussed and refined in Alves et al. (2024), which proposes four classification possibilities: Purely Mathematical, Reality, Semi-reality, and Artificial. In this paper, the focus is on presenting the results that touch on tasks[1] whose contexts were classified as Reality.

2 Exploring Methodological Pathways: Classification and Analysis of Tasks in the New High School Curriculum

This research follows a qualitative approach (Goldenberg, 2011), in which the Theorem System (Amaral et al., 2022) was adopted, as shown in Figure 1, for the textbook view. In order to analyze the works approved by PNLD 2021, during the planning phase, the textbook view was adopted, considering the time available and access. All 10 collections approved by the Program were analyzed, referring to textbook in Mathematics for NEM, totaling 21 textbooks. We chose to analyze the Geometry content of these works, given the familiarity of the research group teorEMa — Interlocutions between Geometry and Mathematics Education with this theme.

During the exploration of the material, the structural characteristics of the textbooks were observed, with special attention to the Student Book and the Teacher's Manual. The latter often contains considerations about the students' work on the tasks, especially those that require the elaboration of activities. Due to the unpredictability of student creativity, the classification of these tasks became unfeasible, and the author's suggestion was followed. The tasks were analyzed in relation to their context.

With regard to data construction, the data were arranged, organized, and quantified in *Google Sheets* spreadsheets, through the implementation of a *Python* algorithm that connects a simple *Google Colaboratory* interface to *Google Cloud*. This feature enables an application programming interface (API) with the spreadsheet, streamlining data construction and

[1] The term *Tasks* will be understood here according to the definition presented in Litoldo (2021).

organization.

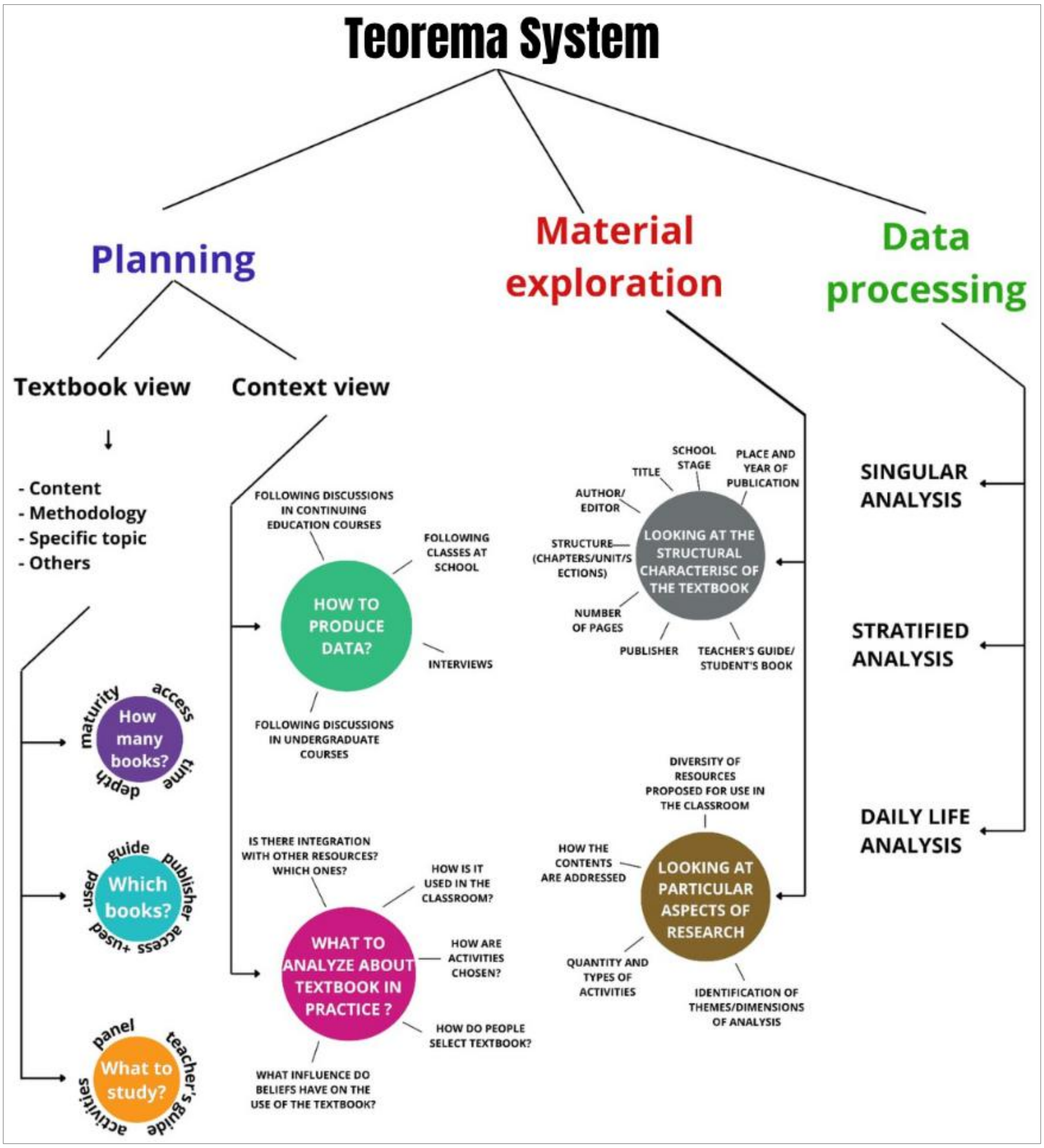

Figure 1: Teorema System (Amaral *et al.*, 2022, p. 202)

The data processing developed followed the stratified analysis approach, in which textbook is considered an object of study in itself and, in this case, the extracts (the common part analyzed in all works) are the proposed tasks, investigating their contexts.

It should be noted that, during the classification of tasks, some procedures were adopted to improve it. The first was the division of the task context into two classifications: the statement and the question, given that, in about 200 tasks, these two contexts diverged. For this reason, we sought to highlight this difference. Tasks that do not fit into either context are classified as *Not applicable*. This type of task corresponds to those proposed in the textbook s, but which cannot be categorized contextually.

3 The different contextual references: building the theoretical framework

To construct the different context references used to classify tasks, we considered what the literature has to say on the subject. Skovsmose (2000) divides them into three: Pure Mathematics, Semi-reality, and Reality. The reference to Pure Mathematics marks an environment in which questions refer exclusively to Mathematics. The reference to Semi-reality presents a constructed/simulated reality, which usually occurs in a superficial manner, failing to encompass reality in a reflective way, so that no other questions about the details of the task context are relevant, only the information necessary for the solution — numerical data. Reality encompasses tasks with references to everyday situations.

Regarding the types of context reference classifications proposed by Dekker and Querelle (2002), three are considered: Virtual Context, Artificial Context, and Mathematical Context. Virtual Context is marked by the presence of elements that are not present in reality but are extracted from it. The Artificial Context is intended to mark scenarios in which the task is in the fantasy world. Finally, the Mathematical Context is characterized by originating from Mathematics itself.

It is understood that the Virtual Context is equivalent to the Semireal proposed by Skovsmose (2000), since both refer to elements extracted from reality. The term Semireal is used, even though Virtual also refers to the same situation. Similarly, the Mathematical context of Dekker and Querelle (2002) and the Purely Mathematical context of Skovsmose (2000) are similar, since both indicate situations contextualized in Mathematics, without relating extra-mathematical elements.

We chose to use Skovsmose's (2000) nomenclature, even though the other terminology designates the same case. Finally, regarding the reference to Artificial Context, there is no equivalent in Skovsmose's (2000) work, as it indicates a context that can be used in tasks. Therefore, in order to avoid missing classifications, it will also be considered in the classification.

Lana and Carrião (2015) present six types of reference classifications, the first three from Skovsmose (2000) and the other three from Duli (2014): Pure Mathematics, Artificial, Reality, Interdisciplinarity, Intradisciplinarity, and History of Mathematics. It is important to note that the latter was not found by the authors in the analyzed work, so they do not delve deeper into its discussion. The Interdisciplinary context is one in which interdisciplinary approaches occur in mathematical tasks in which students are invited to address the dialogue between Mathematics and other subjects in their curriculum. Intradisciplinary, on the other hand, is permeated by the use of contexts that establish the relationship between a given content and other fields of Mathematics itself, intertwining different concepts.

In general, there are some counterpoints in the combination of these two authors' perspectives, because, although they intersect at certain points, they end up generating scenarios that can fit into two or more distinct classifications, which can make the classification process difficult, since the criteria necessary for choosing one over the other are not made explicit.

For example, suppose a scenario that fits what the authors call Intradisciplinarity. If there are no extramathematical elements, it can be classified as Purely Mathematical; otherwise, it may belong to Semireality or even Reality. However, according to Duli (2014), this situation would be characterized as Intradisciplinary. Thus, the clear possibility of double classification is evident.

Furthermore, it is worth noting that, in this context, it is assumed that Mathematics is composed of many interconnected subjects, rather than divided into watertight compartments that students use only while studying a particular subject. This perspective may contribute to a lack of dialogue between mathematical elements on the part of students.

Regarding Interdisciplinarity and the History of Mathematics, the same problem of double classification arises, since when one of these cases occurs, they are related to elements outside Mathematics and, with it, what necessarily falls under the classifications of Semireality

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or Reality, that is, there is ambiguity. Given the above, the union of these types of contexts intersects, which does not contribute to classification, allowing for inaccuracy. Thus, we chose not to use this second part proposed by Lana and Carrião (2015).

Finally, Litoldo (2021) presents a classification of different context references based on three types, derived mainly from Skovsmose (2000) and Dekker and Querelle (2002): Purely Mathematical, Artificial, and Reality. The latter is subdivided into Real and Semi-real; and the second, in turn, is subdivided into two more: Reasonable and Unreasonable. Attention is drawn to an important aspect of this type of context reference, reasonableness, which is understood as the quality of being reasonable:

> Contexts that explore situations or information from semi-reality can be further classified into two groups: i) reasonable, contexts that relate to a possible real-life situation, and ii) unreasonable, contexts that address situations that, although they may exist in everyday life, contain information (mostly quantitative) that is not reasonable to be true in real life. (Litoldo, 2021, p. 115).

In general, the classification developed by Litoldo (2021), illustrated in Figure 2, closely resembles the theoretical framework that supports this research.

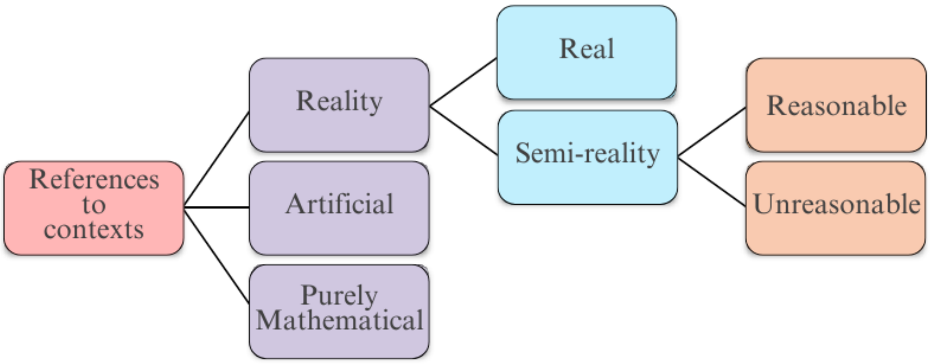


Figure 2: Context and its classes (Litoldo, 2021, p. 145)

In this construction, the division of Reality into Semi-real and Real can generate ambiguities, since the very notion of reality implies its totality, making the existence of a semi-reality within it questionable. For this reason, we chose not to use this separation, even though, at the end of the classification, the differences are insignificant.

On the other hand, the subdivision of Semi-reality contributes to a more accurate classification and highlights a relevant distinction, which deserves to be emphasized in order to promote reflection on student learning. Therefore, this subdivision will be maintained.

Given the caveats and justifications presented, it is pertinent to reflect on the nature of the tasks, especially with regard to the precision of the definitions adopted, with the aim of eliminating ambiguities. Thus, the four main contexts that constitute the theoretical framework will be explored: Purely Mathematical, Artificial, Reality, and Semireality, the latter being divided into Reasonable and Unreasonable. Figure 3 presents a representation of this concept.

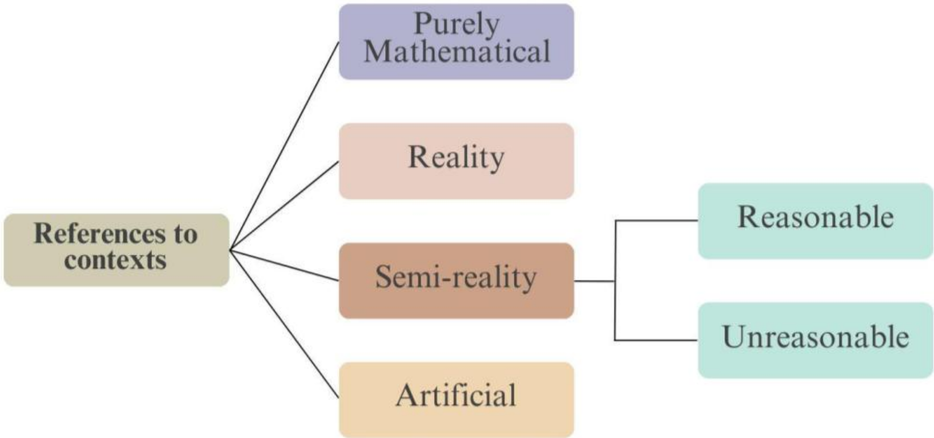


Figure 3: Distinct references to contexts and their classes (Alves, Amaral e Litoldo, 2024, p.10)

3.1 Purely Mathematical

References to purely mathematical contexts are not subjective, referring only to Mathematics and nothing else, and are composed solely of signs, symbols, numbers, figures, and letters (Skovsmose, 2000; Lana and Carrião, 2015; Litoldo, 2021). It should be noted that, according to Dekker and Querelle (2002), the fact that the context is mathematical does not imply that the task is simple, because it addresses a situation involving the use of developed mathematical knowledge, thus encouraging students to think critically and analyze the situation presented (Skovsmose, 2000; Dekker and Querelle, 2002).

3.2 Reality

The contexts referenced in Reality depict scenarios derived from explicit real-life situations, or contain information and quantifications from real sources (Skovsmose, 2000; Dekker and Querelle, 2002; Lana and Carrião, 2015; Litoldo, 2021) or even real images in their statements. References to real-life contexts enable students to produce and develop real concepts and meanings for tasks. Thus, this type of context can boost students' mathematical literacy by encouraging the use of Mathematics in a wide range of situations (Skovsmose, 2000; Dekker and Querelle, 2002; Litoldo, 2021).

In addition, the potential of this context to stimulate the expansion of knowledge stands out, given that it is not limited to the student's daily life, but also refers to facts that are distant from them (Lana and Carrião, 2015; Litoldo, 2021). According to Lana and Carrião (2015), in general, in textbook, these contexts are widely used in conjunction with material published in newspapers or magazines, *websites*, books, or papers. From these sources, authors shape the task in order to establish connections between Mathematics and the real world (Dekker and Querelle, 2002; Lana and Carrião, 2015).

3.3 Semi-reality

The scenarios contextualized in Semireality are marked by the presence of a synthetic reality, that is, created, for example, by the author of a Math textbook based on elements that are close to the student's daily life, but are based on situations that do not emerge explicitly from the real context (Skovsmose, 2000; Dekker and Querelle, 2002; Lana and Carrião, 2015; Litoldo, 2021).

This type of context is widely used when the intention is to address a scenario that, if treated as real, would become more complex. To explore the topic mathematically, the author can modify it to facilitate understanding (Dekker and Querelle, 2002). Furthermore, Litoldo (2021), supported by De Lange (1995), considers this configuration a camouflage context, since it is used only to hide the task that would otherwise be purely mathematical. The reference to the context of Semireality assumed in this work follows the subdivision proposed by Litoldo (2021), which distinguishes two classifications according to their reasonableness: Reasonable and Unreasonable.

3.4 Artificial

Some tasks refer to a context that incorporates non-real elements, linking them to a fantasy world and, therefore, considered an artificial context (Dekker and Querelle, 2002; Litoldo, 2021).

It should be noted that this type of context requires students to be willing to use their imagination to attribute meaning to the task, assuming the proposed situations as real in their minds and experiencing them as if they were actually real (Dekker and Querelle, 2002; Van Den Heuvel-Panhuizen, 2005). It should be noted that, with younger students, especially those in the early years of schooling, these scenarios can contribute to the development of their creativity.

4 Geometry tasks in the context of reality

After analyzing the 21 textbooks that contained Geometry tasks, 7,345 tasks were classified according to the distribution shown in Table 1.

Table 1: Classification of the context of PNLD 2021 tasks

Type of context reference		Statement		Question	
		n	%	n	%
Purely mathematical		4.429	60,30%	4.465	60,79%
Artificial		0	0,00%	0	0,00%
Reality		954	12,99%	793	10,80%
Semi-reality	Fair	1.435	19,54%	1.557	21,20%
	Not Fair	50	0,68%	53	0,72%
Not applicable		477	6,49%	477	6,49%
Total		**7.345**	**100,00%**	**7.345**	**100,00%**

Source: Own elaboration (2025)

It should be noted that, at first glance, the figures in Table 1 reveal two main pieces of information: the absence of tasks with an artificial context, which may indicate a limitation on students' creativity, since they are not being encouraged to use their imagination to make them real in their minds (Van Den Heuvel-Panhuizen, 2005); and the low number of tasks with references to real-life contexts, highlighting the preference of textbooks for purely mathematical scenarios, without subjectivity, or, when present, for simulated situations that did not emerge from real life, which is the case of semi-reality. As discussed in Alves, Amaral and Litoldo (2024, p. 11),

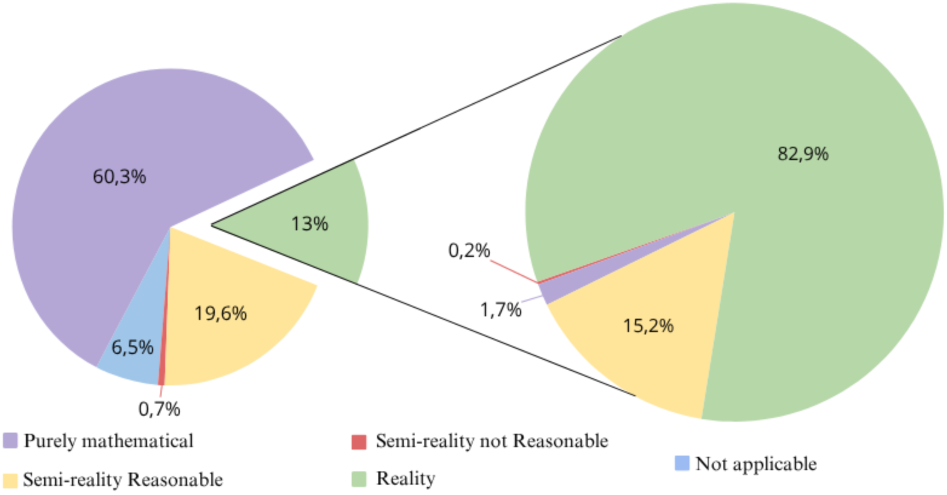
> references to real-life contexts enable students to produce and develop real concepts and meanings for tasks. This type of context can therefore boost students' mathematical literacy by encouraging them to use mathematics in a wide range of situations.

Thus, it can be observed that the presence of only about 13% (954) of the Geometry tasks in the selected works also indicates a lack of connection between Mathematics and real life, both in terms of bringing meaning to student learning and in terms of presenting facts that are distant from their lives through the tasks.

It should also be noted that, although there are a considerable number of tasks whose context reference was classified as Semi-real, this scenario can be considered a camouflage (De Lange, 1995; Litoldo, 2021), since it is often used only to hide a task that would be Purely Mathematical. This characteristic further aggravates the problem of the scarcity of tasks with context references in Reality, since the scenario that would be in between Purely Mathematical and Reality – Semi-Reality – is based on a camouflage of the former.

Furthermore, when analyzing the context of the questions whose statements were classified as Reality, the lack of alignment between the two becomes evident, as illustrated in Graph 1.

Graph 1: Distribution of question contexts in tasks with context in the statement in reality



Source: Own elaboration (2025)

It is worth noting the low presence of references to real-life contexts and, even when this scenario is present in the statement, a considerable portion of the tasks (about 17% – 163 tasks) do not take advantage of this potential in the question. As a result, the positive attributes characteristic of this type of reference may be lost, leading to a reduction in the connection between Mathematics and real life.

This divergence is not exclusive to isolated collections, but is present in all works approved in the PNLD 2021, as shown in Table 2.

Table 2: Distribution of the context of the question in tasks with statements classified in the Reality context

Collection	Reality	Reasonable semi-reality	Unreasonable semi-reality	Purely mathematical
Prisma Matemática (Bonjorno, Giovanni Júnior and Sousa, 2020a, 2020b)	52	4	1	4
Matemática em contexto (Dante and Viana, 2020a, 2020b)	105	38	1	5
Quadrante Matemática (Chavante and Prestes, 2020a,	49	3	0	0

2020b)				
Conexões Matemática (Leonardo, 2020a, 2020b)	36	4	0	0
Diálogo matemática (Teixeira, 2020a, 2020b)	80	47	0	4
Interação matemática (Freitas; Longen and Blanco, 2020a, 2020b)	186	8	1	0
Matemática interligada (Andrade, 2020a, 2020b, 2020c)	55	4	0	2
Matemática nos dias de hoje (Cevada, Silva, Prado and Colpani, 2020a, 2020b)	107	6	0	0
Multiversos (Souza, 2020a, 2020b)	78	27	0	0
Ser protagonista (Smole and Diniz, 2020a, 2020b)	42	4	0	1
Total	**790**	**145**	**3**	**16**

Source: Own elaboration (2025)

In order to explore the possible convergences and divergences between the context of the statement, referenced in Reality, and the questions associated with these tasks, we proposed the analysis of some specific examples. Based on this analysis, we sought to promote an in-depth discussion about the nuances present in each of these tasks, highlighting how the alignment, or lack thereof, between the statement and the question can impact the way students deal with the concepts addressed. Thus, we hope to enrich the understanding of the pedagogical implications of this relationship in the teaching and learning process of Geometry.

4.1 Double reality: aligning statements and questions

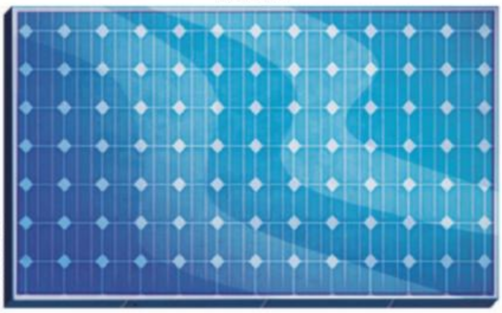
Tasks whose context has been classified as Reality "depict scenarios derived from explicit real-life situations, or contain information and quantifications from real sources, or even real images in their statements" (Alves, Amaral and Litoldo, 2024, p. 11). This type of context reference allows students to assign real meanings to tasks, which can boost their mathematical literacy and encourage the use of Mathematics in a wide range of situations. Furthermore, this approach favors the expansion of students' knowledge, as it is not restricted to their daily lives, but can also refer to facts that are distant from their reality.

In the task statement (Figure 4), there is a mention of photovoltaic energy production in homes. The three questions linked to the statement invite students to explore this topic further by investigating energy consumption in different public places and designing a project for the installation of solar panels in these spaces. To do so, they must consider factors such as the efficiency of this system and the number of photovoltaic cells needed for its proper functioning, which involves the concept of area.

39. Sunlight is an alternative source for generating electricity. In ordinary homes, it is possible to install an autonomous photovoltaic energy production system, which generates electricity from sunlight using solar panels.

> The solar panel has a rectangular shape and is composed of photovoltaic cells, a device responsible for capturing sunlight and converting it into electrical energy used in the home.

a) Rectangular solar panels of the model shown below are being installed on the roof of a family home. For this installation, it is estimated that each square meter of the panel will generate an average of 21 kWh of electricity per month.



FABIO EUGENIO

> Power of each panel: 270 W.

- The energy efficiency of a solar panel corresponds to the percentage of energy from sunlight that is converted into electrical energy per square meter. To determine this efficiency, we divide its power (W) by the area (m^2) and then divide the result by 10. Calculate the approximate energy efficiency of each solar panel that will be installed in this residence.
  16.36%
- In this residence, the average monthly electricity consumption is 275 kWh. At a minimum, how many solar panels need to be installed on the roof (in order to supply all this consumption)? What will be the area occupied by all these solar panels?
  8 solar panels 13.2m².

b) Consult a bill to find the average monthly electricity consumption of your residence. Then determine how many panels of the model presented in item a are needed to be installed to supply all this consumption.
Personal answer.

c) Get together with two classmates and develop a proposal for the installation of solar panels for electricity generation in a local public building, such as your school, a basic health unit, the headquarters of the neighborhood residents' association, etc. To develop this proposal. Investigate the usual electricity consumption of this space, research the efficiency of available solar panels and the cost of installation. With this information, calculate the area of panels needed to supply the demand for electricity. Sketch a possible location where these solar panels could be installed, such as a roof or open area. Finally, write an official report presenting arguments, based on reliable data, that support this installation, both for environmental and financial reasons.
Personal answer.

PEDRO RIBAS/SMCS/PREFEITURA DE CURITIBA

> Solar panels installed on the roof of the Curitiba city hall (Palácio 29 de Março). Photograph from 2019.

Figure 4: Example of a statement and question classified in reality (Souza, 2020a, p. 33)

The depth provided by these tasks allows new questions to emerge, demonstrating the complexity and richness of the real-life situations involved. By questioning issues such as calculating the area of irregular roofs, the variation in the efficiency of solar panels in different locations, or the financial impact of solar energy projects, students are challenged to apply mathematical concepts in scenarios that connect Mathematics to social, economic, and environmental issues. These problems not only broaden critical thinking, but also promote a more systemic understanding, leading students to consider interdisciplinary variables and seek solutions that integrate different areas of knowledge, such as geography, economics, and engineering.

However, this multiplicity of possibilities also reveals the pedagogical complexity involved. Teacher guidance becomes essential to guide students in an investigation that often does not have a single or definitive answer. Breaking with the traditional teaching model, in which there is a predefined correct answer, requires a new attitude from teachers, who need to deal with the uncertainty and unpredictability of learning, while encouraging students to develop intellectual autonomy.

In addition, the reflections raised by these tasks open space for ethical and political discussions, addressing, for example, the feasibility of certain energy projects for a city or country, the environmental impacts of energy sources, and inequalities in access to technological resources. In this way, Mathematics is redefined as an essential tool for the critical analysis of global and local problems. Thus, these tasks promote a form of teaching that transcends mere technical application, engaging students in deep reflection on the role of Mathematics in understanding and transforming reality.

4.2 The contextual paradox: when the real meets the abstract

Tasks with a purely mathematical context reference are marked by the absence of subjectivity, "referring to mathematics and only mathematics, being permeated only by signs, symbols, numbers, figures, and letters" (Alves, Amaral and Litoldo, 2024, p. 10). Given this, there is a certain contradiction in their use together with references to contexts in Reality, since they are almost antonymous, as the main characteristic of one is the connection between Mathematics and the real world, while the other involves only elements related to Mathematics. It is also worth noting that the possibility of questioning the information presented, a determining characteristic of this situation, is lost in the exclusive use of Mathematics, which presupposes, above all, a single correct answer in most cases, which is not always the case in references to the context of Reality.

Figure 4 is marked by the dialogue between Mathematics and real life, relating the construction of a building to some concepts of Geometry. This situation can be explored in several ways, such as asking students to reflect on the possible advantages of using triangles in construction, also asking them to look for this shape around them and understand why it is used in these situations. However, when paying attention to the characteristic of the question (task item b), there is a surprise: even without encouraging students to reflect on the presence of triangles in construction, the textbook asks them to identify which of the triangles are right triangles according to their side measurements, ignoring the context presented above.

Thus, it can be seen that the wording of the question in this task (item b) may not be related to the statement, that is, the context presented previously is not effective, but only attempts to camouflage a task that would be purely mathematical. It can be observed that, with this articulation, the striking characteristics of the context references in reality are lost. Furthermore, apart from the connection with real life, the information presented can no longer be questioned: what is the purpose of identifying right triangles? Are they present in any construction? How do engineers and architects articulate these shapes when designing their projects? Why are triangles chosen? Why must they be right-angled?

This misalignment between the context of the statement and the question, illustrated in Figure 5, raises important questions about pedagogical coherence in the use of contextualized tasks from Reality. The task, which initially seemed promising by connecting Geometry to a concrete situatio, the construction of a building, loses its educational potential when the question deviates from this context, becoming a purely mathematical task. This disconnect not only weakens the original contextualization, but also reduces the opportunity for students to critically explore the relationship between geometric concepts and the real world.

12 The construction of a building requires meticulous work, from the design phase to its execution. One of the stages of the construction process consists of contracting the building, that is, marking the axes of its walls and other structural elements on the land, as stated in the project. During the transaction, it is essential that the actions are carried out as precisely as possible, to avoid errors that could compromise the construction as a whole, since, for example, the positioning of the foundation elements used – such as piles driven into the ground – will serve as the basis for the execution of its entire structure.

Workers at the building rental stage.

In view of this, it is extremely important to check, for example, the orthogonal axes marked at the end of each stage of the location. For this check, topography equipment can be used or, even in a simple way, the right-angled triangle method, through which it is possible to check whether these axes form a angle.

a) In civil construction, the principle of the right-angled triangle mentioned consists of the idea of forming triangles with sides of 3 m, 4 m and 5 m in length, for example. Why can a triangle with side lengths equal to 0.6 m, 0.8 m and 1 m also be used to check right angles?

b) Among the figures presented, which are right-angled triangles?

Triangles ABC and MNO.

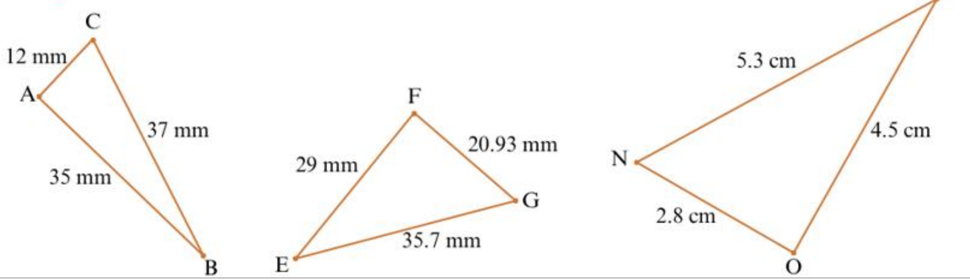


Figure 5: Statement classified as Reality and question in Purely Mathematical (Teixeira, 2020a, p. 27)

By focusing only on identifying right triangles from measurements, without any explicit connection to the scenario presented, the question (item b) ignores the potential for reflection that the statement offered. Questions such as the function of triangles in architectural structures, the role of engineers and architects in choosing these shapes, and even the geometric advantages of triangles in construction could enrich students' understanding of the practical application of Mathematics in everyday life.

Furthermore, this misalignment between the statement and the question reveals a limitation in the way the tasks are structured. The goal of promoting a connection between Mathematics and everyday life, which should be the central focus of tasks that are fully contextualized in reality, is compromised. This deprives students of the chance to question and investigate deeper aspects of the application of Geometry in the world. Instead of simply identifying right triangles, they could be asked to question why these specific shapes are used in construction and how their choice can impact aspects such as stability or architectural design. This type of critical reflection is essential for developing a more robust and integrated understanding of Mathematics.

Finally, although tasks with a purely mathematical context are essential for students to understand the internal structure of this science, it is equally important for them to experience Mathematics in real-life contexts, where concepts take on practical meaning. The link between formal learning and everyday life brings new value to mathematical concepts, making them more accessible and relevant to students. When this connection is lost, as in task b in Figure 4, Mathematics becomes distant from students' lives, which can reduce their ability to see the usefulness of the concepts they have learned, weakening their engagement and motivation to learn.

4.3 Disconnections from Reality: the incoherent semi-real context

The tasks referenced in Unreasonable Semi-Reality are marked by the articulation of a simulated reality with mathematical concepts, "however, they contain information that conflicts with reality, and therefore, there is incorrect data in their elaboration" (Alves, Amaral and Litoldo, 2024, p. 14).

Figure 6 illustrates two tasks included in the section For reflection and discussion which, according to the material provided to the teacher:

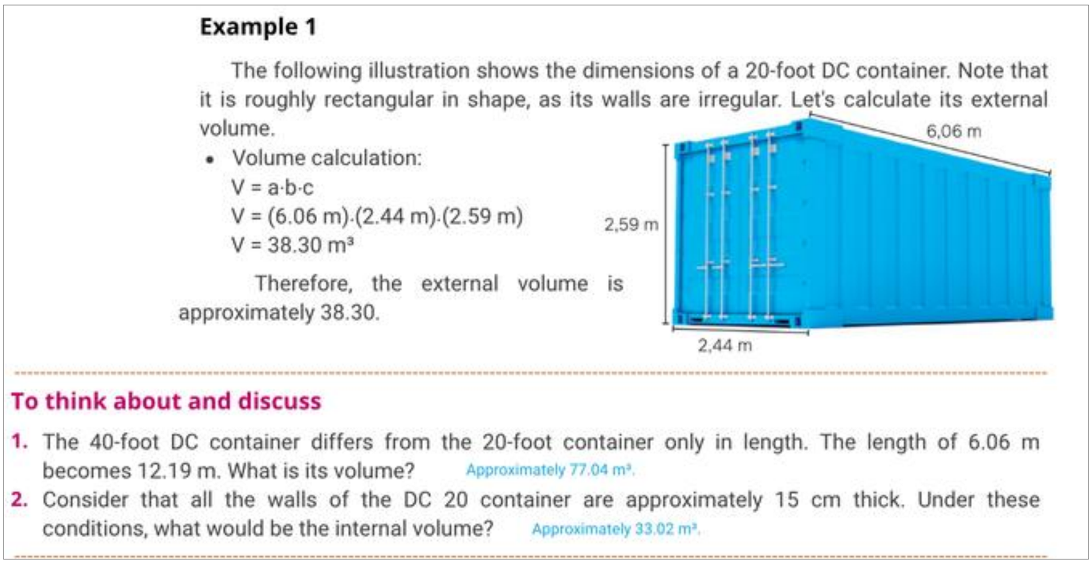
> The objective [of the section] is to place students as active participants in their own learning process, systematically enhancing their ability to make inferences by improving their reading skills. It consists of situations that lead students to think individually, either during class or as homework. The questions posed may involve investigating properties, developing new problems, drawing conclusions, or synthesizing ideas that can be presented through an argumentative text or mathematical solutions to justify the arguments presented in the discussion, as appropriate, which provides another opportunity for students to develop their argumentation skills. (Freitas, Longen and Blanco, 2020, p. 31).

Note that, prior to the tasks, the material focuses on presenting *containers*, illustrating an international cargo transport model, the DC 20. The presentation of the DC 20 container initially seeks to contextualize the task in the cargo transport scenario, providing a practical example of the application of mathematical concepts. However, when looking at the details of the question, inconsistencies arise that compromise the validity of the task. The statement that the container walls are 15 cm thick is unrealistic, as the walls of this type of container are made of corrugated steel with a standard thickness of approximately 14 mm[2]. This discrepancy, which multiplies the actual thickness by a factor of almost 11, has serious implications for the tasks. If this thickness were true, the weight of the container would increase significantly due to the greater amount of material, especially considering the density of steel. This increase in weight would cause major logistical challenges, such as transportation difficulties and higher operating costs, making the proposed example unfeasible in reality.

This change in situation leads us to a semi-reality, in which the task loses meaning, since the information provided is inconsistent with the real context previously presented. This situation is characterized as an example of unreasonable semi-reality, in which the information presented to students does not reflect reality. The most worrying thing is that students may assume that this incorrect data is true, compromising the credibility of the tasks and the learning

[2] Information taken from Miranda Container (https://mirandacontainer.com.br/container-de-20pes-6-metros/).

process. The purpose of the section in which these tasks are presented is precisely to encourage debate and critical reflection, allowing students to make the connection between Mathematics and reality. When this information is not properly considered, inappropriate situations arise. Instead of encouraging questioning and critical analysis, this type of task can create misconceptions in students, affecting the way they view both Mathematics and the real world.



Example 1

The following illustration shows the dimensions of a 20-foot DC container. Note that it is roughly rectangular in shape, as its walls are irregular. Let's calculate its external volume.

- Volume calculation:

V = a·b·c
V = (6.06 m)·(2.44 m)·(2.59 m)
V = 38.30 m³

Therefore, the external volume is approximately 38.30.

To think about and discuss

1. The 40-foot DC container differs from the 20-foot container only in length. The length of 6.06 m becomes 12.19 m. What is its volume? Approximately 77.04 m³.
2. Consider that all the walls of the DC 20 container are approximately 15 cm thick. Under these conditions, what would be the internal volume? Approximately 33.02 m³.

Figure 6: Statement classified as reality and question classified as unreasonable semi-reality (Freitas, Longen and Blanco (2020a, p. 62)

As Skovsmose (2000) points out, Mathematics in investigative contexts should foster the development of *critical thinking*, allowing students to question the data and contexts presented. When information is inconsistent or inaccurate, the potential of the task is compromised, and it fails to fulfill its role of connecting Mathematics with everyday life, distancing students from a practical and meaningful understanding of concepts. Instead of just solving tasks mechanically, contextualizing them in Semireality in a reasonable way should give students the opportunity to think critically about the information they receive, connecting mathematical learning to real and plausible issues.

4.4 Between Reality and Simulation: the Reasonable Semi-Real Context

The Reasonable Semi-Real context is marked by:

> [...] presence of a synthetic reality, that is, created, for example, by the author of a mathematics textbook based on elements that are close to the student's everyday life, but are based on situations that do not emerge from the real context, at least not explicitly (Alves, Amaral and Litoldo, 2024, p. 13).

Furthermore, its reasonableness concerns the possibility of the scenario created being witnessed in reality (Litoldo, 2021).

Figure 7 shows a task whose statement was classified as belonging to the Reality context, while its question falls under the Reasonable Semi-Reality classification. The task development uses elements taken from the real world, establishing a connection between a typical Japanese dish, temaki, which is cone-shaped, and mathematical concepts. However, the question simulates a semi-real situation in which temaki is used as a basis for density and volume calculations, partially distancing itself from the concrete reality of the students.

22. A Japanese dish, temaki is a type of sushi in the shape of a cone, rolled externally with nori, a type of sheet made from seaweed, and stuffed with rice, raw fish, fish eggs, vegetables and a paste of mayonnaise and chives.

A typical temaki can be represented mathematically by a straight circular cone in which the diameter of the base measures 8 cm and the height is 10 cm. Knowing that, in a typical salmon temaki, the fish corresponds to 90% of the mass of its filling, that the density of salmon is 0.35 g/cm^3, and taking $\pi = 3$, the approximate quantity of salmon, in grams, in this temaki, is

Alternative d.

a) 46.
b) 58.
c) 54.
d) 50.
e) 62.

Density (d) is defined as the ratio between the mass (m) and the volume (v) of a body or substance, that is, $d = \frac{m}{v}$.

Figure 7: Statement in reality and question in reasonable semi-reality (Chavante and Prestes, 2020a, p.134)

It is important to highlight that the potentialities related to context reference in Reality were largely lost in this task. Although the proposal allows students to work with seemingly realistic information, such as the shape and physical characteristics of temaki, the suggested calculation does not reflect a plausible situation that they would encounter in everyday life. Instead of promoting a concrete connection between Mathematics and the real world, the task veers toward simplification, in which temaki is used instrumentally, without encouraging students to critically question the assumptions involved.

Furthermore, by stating that this is a *typical temaki*, the task fails to adequately contextualize what is meant by *typical*. Mathematics, in this case, is dissociated from the cultural diversity and practical variations that could enrich students' reflection. A simple internet search reveals that different cultures and regions have significant variations in the measurements and compositions of temaki, which leads us to question: to whom does this “typical temaki” refer? Is it typical for Japanese, Brazilians, or Europeans? By not addressing these issues, the task is limited to a generic context, missing the opportunity to explore the cultural and geographical implications that could enrich the mathematical discussion.

Thus, the lack of clarity about what constitutes a *typical temaki* and the distance from the reality experienced by students reduce the opportunity for critical reflection on the real context of the task. Instead of promoting a richer exploration connected to the world around them, the task remains in a didactic comfort zone, where students perform calculations without reflecting on the real meaning of the information provided. A more critical and culturally informed approach could stimulate deeper reflection on how Mathematics applies differently in different cultural contexts.

It is observed that this type of context does not encourage these types of questions, but is structured to be passively accepted by students. It is irrelevant who will consume the food or who will prepare it, as the sole purpose of the task is to be solved mechanically, exemplifying what De Lange (1995) calls *camouflage contexts*. These contexts appear to relate to reality, but in fact hide the absence of a deeper engagement with the concrete world. Therefore, the connection between a situation that allows questions such as those in Reality, and a context prepared only to be solved with a single correct answer, represents, once again, the loss of the potential offered by different contextual references, especially in the works of NEM.

5 Final thoughts

This research revealed a pressing need to reflect on the quality of the tasks proposed in the PNLD works, especially with regard to the contextual references that students are experiencing. The data highlight the lack of alignment between the statement and the question in the tasks, particularly in the context of Reality. It is noted that part of the power of this context is lost by not giving students the opportunity to explore issues that actually make them reflect on real solutions.

The low presence of tasks that are fully contextualized in Reality — that is, those in which there is alignment between the statement and the question — evidenced by the analysis of the works, reflects a worrying gap in the opportunities offered by the tasks present in the textbooks, in the field of Geometry. In many situations, the preference for purely mathematical contexts or semi-realities that camouflage the true purpose of the task transforms the students' mathematical experience into a mechanical practice, disconnected from the real world. When disconnected from the reality experienced by students, Mathematics can lose some of its ability to engage and provide learning based on reality.

In addition, tasks that explore Semireality contexts, especially unreasonable ones, pose an additional risk to teaching. By using unrealistic data or situations without providing opportunities for questioning or critical reflection, these tasks not only fail to promote a deeper understanding of mathematical concepts, but can also lead students to form misconceptions about the applicability of Mathematics in the real world. The challenge, therefore, is to ensure that Semireality tasks promote a balance between pedagogical simplification and verisimilitude in order to preserve the potential for critical reflection on the part of students.

Additionally, analyses indicate that the absence of tasks that explore Real contexts in depth deprives students of a richer and more challenging experience in the field of Mathematics. Solving tasks involving real-world contexts, such as solar energy production or building construction, for example, could expand students' knowledge beyond the technical sphere, connecting mathematical concepts to social, economic, and environmental issues. When Mathematics is worked on through different contextual references, it can take on new meaning for students, who are privileged to work in diverse situations, thereby complementing certain gaps in their learning.

In conclusion, this research highlights the need for greater rigor in the design of tasks that truly integrate real-world contexts into PNLD teaching materials. For Mathematics to fully fulfill its educational role, it is essential that teaching materials offer students the opportunity to explore the world around them, critically reflecting on the real problems they face.

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Conflicts of Interest

The authors declare no conflicts of interest that could influence the results of the study presented in the article.

Data Availability Statement

The data collected and analyzed in the article will be made available upon request to the authors.

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