

Cryptography: a didactic possibility for teaching matrices

Abstract: This paper presents the development of a teaching sequence that combines the topic of cryptography with the content of matrices, using spreadsheets. Based on the ideas of curriculum integration, topics of interest, Contemporary Transversal Themes, and the use of technologies, we seek to present a possible path for teaching practice. The objective was to explore the potential of this approach to promote contextualized teaching. The qualitative research was developed with 23 students from a public school in Rio Grande do Sul, using questionnaires and written records to produce data. The results indicate that the teaching sequence contributed to the understanding and review of matrix content, enabling contextualization and the use of technological resources.

Keywords: Cryptography. Curriculum. High School. Mathematics.

Criptografia: una posibilidad didáctica para la enseñanza de matrices

Resumen: Este artículo presenta el desarrollo de una secuencia didáctica que combina el tema de criptografía con el contenido de matrices, utilizando hojas de cálculo electrónicas. A partir de las ideas de integración curricular, temas de interés, Temas Contemporáneos Transversales y el uso de tecnologías, buscamos presentar un camino posible para la práctica docente. El objetivo fue explorar el potencial de este enfoque para promover la enseñanza contextualizada. La investigación cualitativa se desarrolló con 23 estudiantes de una escuela pública de Rio Grande do Sul, utilizándose cuestionarios y registros escritos para la producción de datos. Los resultados indican que la secuencia didáctica contribuyó a la comprensión y revisión del contenido de la matriz, posibilitando la contextualización y el uso de recursos tecnológicos.

Palabras clave: Cifrado. Currículo. Escuela Secundaria. Matemáticas.


Criptografia: uma possibilidade didática para o ensino de matrizes

Resumo: Este artigo apresenta o desenvolvimento de uma sequência didática que alia o tema criptografia ao conteúdo de matrizes, utilizando planilhas eletrônicas. Com base nas ideias de integração curricular, temas de interesse, Temas Contemporâneos Transversais e o uso das tecnologias, busca-se apresentar um caminho possível para a prática docente. O objetivo foi explorar as potencialidades dessa abordagem para promover um ensino contextualizado. A pesquisa qualitativa foi desenvolvida com 23 estudantes de uma escola pública no Rio Grande do Sul, utilizando questionários e registros escritos para produção de dados. Os resultados indicam que a sequência didática contribuiu para a compreensão e revisão do conteúdo de matrizes, possibilitando a contextualização e o uso de recursos tecnológicos.

Palavras-chave: Criptografia. Currículo. Ensino Médio. Matemática.

1 Introduction

Research in Mathematics Education highlights the importance of a contextualized approach to teaching mathematical content in Basic Education. Thus, teaching through themes relevant to students' education is a valid strategy for developing mathematical content, as it allows teaching to be contextualized based on real-world situations.

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ARTICLE

The *Base Nacional Comum Curricular* [National Common Curriculum Base — BNCC] uses the strategy of teaching with themes, with the proposal of Contemporary Transversal Themes (Brasil, 2019), as well as reiterating the importance of contextualizing content in order to contribute to the critical and civic education of students. From this perspective, the Themes of Interest proposed in Olgin (2015) encompass different characteristics and relationships with the mathematical content of high school that can connect with students' daily lives. Among the themes classified in Olgin (2015), Technological Knowledge and Contemporaneity stand out for this article, highlighting the relevance of digital technologies and issues arising from social contexts.

Thus, cryptography emerges as an appropriate theme for exploring these Topics of Interest, as it enables the contextualized teaching of mathematical content such as arithmetic, combinatorial analysis, statistics, functions, matrices, and others. It also provides opportunities for the use of digital technologies as a facilitating tool in solving problems involving numbers and codes. In this way, teaching activities involving encoding and decoding can be proposed to explore mathematical content, with a view to improving the concepts studied and attributing meaning to student learning (Olgin, 2015).

In this article, we present the contributions of a teaching sequence¹ that was developed based on the theme of cryptography to work on the content of matrices in High School using Excel spreadsheets.

2 Mathematics curriculum and teaching through themes

From Coll's perspective (1999), the curriculum can be conceived as a plan that guides educational practices, defining objectives and providing support for teaching activities structured around central questions, such as what to teach, when to teach, how to teach, and how and when to assess.

Expanding this view, D'Ambrosio (2011) understands the curriculum as a set of strategies that aim to achieve broad educational goals, developed from the integration of objectives, content and methods. Sacristán (2017) emphasizes that the curriculum should not be seen as a static object or as the result of a coherent and definitive educational model, but as a praxis that transcends the explicit dimension of the cultural socialization project in schools, a socializing and cultural practice expressed through various subsystems and pedagogical practices.

In research, the curriculum is understood as an essential element for the organization and orientation of educational practices, functioning as a strategic, normative, and practical tool. It is not limited to being a prescriptive document, but represents a dynamic and constantly evolving process that integrates educational objectives with pedagogical and social practices in a comprehensive and flexible manner. Thus, the curriculum is understood as a social and practical construction that, among other functions, selects and legitimizes knowledge.

Regarding the Mathematics curriculum, Azcárate (1997) argues that it should be structured through a network of themes that allows students to understand and interact with social, cultural, political, and economic reality. Thus, the content involved should reflect the interests, concerns, and challenges of students, being connected to different aspects of everyday life. Beane (2003) complements this perspective by suggesting that knowledge should be organized around organizing centers — relevant problems or central themes — that seek to

¹ This article is an excerpt from the master's dissertation entitled *Paths to the High School Mathematics curriculum: contextualizing matrix content with the theme of cryptography*, defended in the Postgraduate Program in Science and Mathematics Education (PPGECIM) at the Lutheran University of Brazil, written by the first author and supervised by the second author.

relate the curriculum to real-world issues, promoting the unification of often fragmented knowledge. For the author, these organizing centers are used to develop activities that integrate different content and disciplines, demonstrating to students that school knowledge is directly linked to their experiences and needs outside of school.

Thus, the authors emphasize the importance of a curriculum that is deeply connected to the social context of students, promoting integrative learning. In this view, school knowledge is not treated as isolated elements, but interconnected with the reality experienced by students, favoring a contextualized education that is relevant to their comprehensive education.

The BNCC emphasizes the importance of educational institutions incorporating contemporary themes into their curriculum (Brasil, 2018). To this end, the regulation highlights that, when working with these subjects, it is necessary to consider the needs, possibilities, and interests of students, in addition to their linguistic, ethnic, and cultural identities. In this context, Contemporary Transversal Themes (TCT) are introduced, which aim to contextualize the content taught, bringing up topics of interest to students that are relevant to their development as citizens (Brasil, 2019).

Thus, the TCT address issues related to the experiences of the school community and contemporary life, and can be worked on in a cross-curriculum and integrative manner in various subjects. By developing skills associated with the curriculum components, the TCT are considered essential content for Basic Education and are organized into six macro-thematic areas that encompass fifteen contemporary themes (Figure 1).

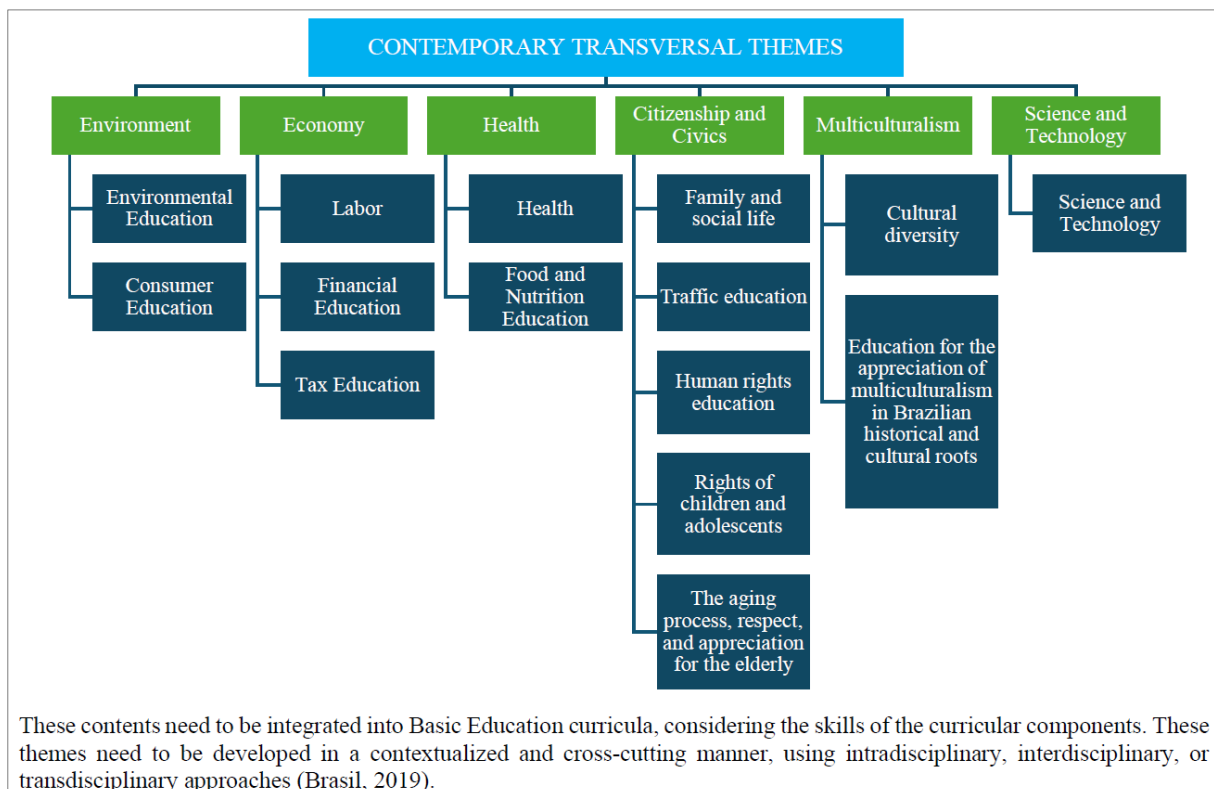


Figure 1: Contemporary Transversal Themes of the BNCC (Adapted from Brasil, 2019)

The TCT cover topics related to real-world issues, such as the use of money and digital technologies, health care and the environment, respect for differences, civil rights and duties, among others (Brasil, 2019).

In line with the proposed work through themes, the Themes of Interest (Olgin, 2015) stand out, enabling students to develop social, cultural, political, and economic values, meeting

the needs and objectives of those involved in the educational relationship. In Olgin (2015, p. 65), these themes are defined as “subjects relevant to student development, modern themes that can enhance the High School Mathematics Curriculum, allowing for the development of mathematical content”.

A classification of these subjects into eight Themes of Interest (Table 1) with the aim of promoting in the Mathematics curriculum “a Critical, transformative, reflective education, rich in contexts, allowing students to engage with each subject in order to review, deepen, exercise, and study the contents of the Mathematics Area” (Olgin, 2015, p. 130).

Table 1: Classification and description of topics of interest

Topics	Description
Technological Knowledge	This theme enables work with Information and Communication Technologies (ICT), as it can assist and enrich the teaching and learning process by facilitating research, information exchange, and simulations of real environments, among other things.
Contemporaneity	This theme enables students to become involved in a network of subjects that allows them to interact with the content, showing its applicability in today's society.
Culture	This theme allows the development of subjects related to music, theater, visual arts, and sports, considering aspects related to local traditions in which students are immersed.
Intramathematics	This theme explores mathematical topics that have been developed throughout history and have led to the development of both mathematics and several other areas. They can also be a resource that promotes the development of logical-mathematical thinking.
Environment	This theme allows students to perceive the interrelationships between the environment and the natural and social world, based on local (culture and tradition) and scientific knowledge.
Political-social	This theme deals with issues relevant to the formation of students as critical, reflective, and socially committed individuals, as it addresses issues related to reality, student interests, and the rights and duties of citizens.
Health	This theme develops topics that, when related to mathematical content, can help students in their way of life and in choosing habits that are relevant to a healthy life.
Local Topics	This theme allows mathematical content to be related to topics from the reality in which students live, enabling discussions on issues related to social practices and local conflicts.

Source: Kranz (2021, p. 48)

The proposal presented in Olgin (2015) aims to include topics related to contemporary life, connecting them to mathematical content, with the goal of identifying the possibilities and challenges for their implementation in the High School Mathematics curriculum. Among the topics of interest, Technological Knowledge and Contemporaneity stand out, enabling work with digital technologies and the development of current issues in society, such as cryptography.

Due to its applicability and the contextualization of mathematical content, such as matrices, the topic of cryptography was developed in this research by exploring the resources of Excel spreadsheets through a didactic sequence.

3 Digital technologies: the use of spreadsheets in the classroom

The BNCC mentions the importance of incorporating digital technologies into school curriculum with the aim of promoting improvements in education, given that, by understanding and using these resources, students can communicate, access, and share information, as well as produce knowledge (Brasil, 2018). In this context, one of the objectives of mathematics in secondary education is to use digital technologies, such as calculators and spreadsheets, to describe and represent situations and characteristics of reality mathematically (Brasil, 2018).

For digital technologies to contribute effectively to the knowledge-building process, they need to be incorporated and integrated into educational processes; otherwise, their transformative potential will not be fully exploited (Valente, 2013). In this sense, Moraes and Fagundes (2011) point out that the use of this resource should enrich the school environment and promote the exchange of knowledge among students. For Castro (2016), digital technologies are powerful allies in mathematics teaching, providing new ways of perceiving content that could not be developed with traditional technologies alone. In addition, these tools enable the use of different languages and means of representing knowledge.

Regarding the use of spreadsheets as a teaching resource, the *Orientações Curriculares para o Ensino Médio* [Curriculum Guidelines for High School — OCEM] indicate that spreadsheets are programs that manipulate tables whose cells can be related by mathematical expressions (Brasil, 2006). Furthermore, to operate a spreadsheet at a basic level, mathematical knowledge similar to that required for using calculators is necessary, but with greater criteria regarding notation, since operations and functions are defined on the cells of a table, using notation for matrices (Brasil, 2006).

Furthermore, Flores (2021) highlights that the use of spreadsheets favors the catalysis of changes in mathematics classes, as the software can act as a conduit for creativity, facilitating simulations and modeling of everyday life. For Conceição (2013), spreadsheets allow calculations to be automated and transformations in the plane to be investigated. In Dellenghausen, Galle, and Olgin (2017), spreadsheets are understood as facilitators of mathematics teaching, allowing students to discover commands and formulas, verify results, and expand their knowledge.

The use of digital technologies in the classroom can enhance the teaching and learning process due to the different possibilities for knowledge construction that they offer. In this sense, we sought to explore the resources of Excel spreadsheets in the teaching sequence, aiming to familiarize students with these technologies, present their potential for teaching mathematics, and the commands necessary for using this tool.

4 A brief history of cryptography

Cryptography refers to the art or science of writing in code, that is, the concealment of messages to ensure the security of their transmission (Carneiro, 2017; Urgellés, 2018). The development of methods for concealment was crucial to ensure that messages were transmitted securely and efficiently, without their content being intercepted.

From this need arose codes, encryptions, and keys, which are fundamental elements of cryptography. Historically, wars have played an important role in the advancement of cryptography, as secure communication between allies was essential. In the 5th century BC, the Spartans used a device known as the Citale to send messages during battles against the Athenians.

This device consisted of a stick on which strips of leather or paper were wrapped and the message was written along the stick. The cryptographic method used by the Citale was

based on transposition, which rearranged the characters of the message (Urgellés, 2018).

One of the first cryptographic substitution methods was developed by Polybius in the 3rd century BC and consisted of replacing the letters of the message with other letters or symbols. In Polybius' encryption, each letter was represented by a pair of letters. Later, the Caesar encryption, created in the 1st century BC, replaced each letter of the message with another that was three positions ahead in the alphabet (Urgellés, 2018). These substitution methods, known as monoalphabetic encryptions, were relatively simple and could be easily deciphered by cryptanalysts through frequency analysis.

The polyalphabetic encryption method was introduced by Leon Battista Alberti in the 15th century, which used two encrypted alphabets. However, in the 16th century, the Vigenère encryption became the most famous of these encryptions, allowing messages to be encrypted based on a keyword and 26 encrypted alphabets (Singh, 2003; Urgellés, 2018). In the 17th century, the Gronsfeld encryption used only 10 alphabets and, as a key, digits from 0 to 9. Later, in the 19th century, Wheatstone and Lord Playfair developed a variation of the Polybius encryption, known as the Playfair encryption, which replaced pairs of letters with other pairs (Singh, 2003; Urgellés, 2018).

During World War I, Germany used the ADFGVX encryption, which combined substitution and transposition methods to encode its messages (Singh, 2003; Urgellés, 2018). And in World War II, the Enigma Machine, invented by Arthur Scherbius in 1923, was a powerful encryption tool used by Nazi Germany. However, in 1939, a team of cryptanalysts led by Alan Turing managed to decipher the Enigma, developing the prototype of the first modern computer, the Colossus (Singh, 2003; Urgellés, 2018).

Seeking new cryptographic methods, in 1929, mathematician Lester Sanders Hill developed a system based on modular arithmetic and linear algebra, using matrices. However, the greatest advance in cryptography came with modern computing, where binary language allowed for the fast and efficient performance of cryptographic tasks. According to Urgellés (2018), computer cryptography began to evolve in the 1970s with the Lucifer algorithm, designed to protect messages sent by computers. The public key algorithm, initially theorized by Whitfield Diffie, was perfected in 1977 by Ron Rivest, Adi Shamir, and Len Adelman, who developed the RSA algorithm, based on the properties of prime numbers, which is still widely used today (Benatti and Benatti, 2019).

However, modern cryptography may face challenges such as the advancement of quantum computing, which has the potential to process information exponentially faster than computers specifically. In 1984, Charles Henry Bennett and Gilles Brassard devised a quantum cryptography system based on the transmission of polarized photons, demonstrating the predictions of this new cryptographic approach (Urgellés, 2018).

Rodrigues and Sá (2019) argue that cryptography is a generative topic that can facilitate mathematical learning because it is present in students' daily lives, even if implicitly, and because it enables constructive and enjoyable engagement. Thus, it is possible to use both the historical approach, which shows the evolution of cryptography over time, and connect it to the mathematical content explored in high school, promoting a more dynamic and contextualized teaching sequence.

5 Methodology

The research adopted a qualitative approach, whose objective was to describe the meaning of the results obtained through questionnaires, written records, and files of the activities carried out by the participants, without resorting to quantitative measurement. Descriptive analysis of the data obtained was also used to provide a broader and more

meaningful understanding of the phenomenon studied (Bogdan and Biklen, 1994).

In order to enhance the teaching and learning of mathematical content, a didactic sequence was developed that employed technological resources combined with the theme of cryptography, aiming at the development of matrix content, as guided by national curriculum documents (Brasil, 2018, 2019). The research was developed in six stages, described below.

Stage 1 — survey of research in papers, dissertations, and theses in the field of mathematics education that addressed the topic of cryptography related to high school mathematics content;

Stage 2 — study of Brazilian curriculum documents and the approach of research and textbooks from the *Programa Nacional do Livro e do Material Didático* [National Textbook and Didactic Material Program — PNLD], 2018 edition, on the content of matrices;

Stage 3 — development of a theoretical framework around Topics of Interest for the High School Mathematics curriculum, Technologies in Mathematics Education, and a historical overview of the topic of cryptography;

Stage 4 — construction of questionnaires and a teaching sequence that included the comic book *Aurora e a criptografia* [Aurora and Cryptography], activities involving historical encryptions and matrix content, and videos;

Stage 5 — application of questionnaires and the didactic sequence with 3rd-grade High School students from a public school in the municipality of Montenegro/RS, remotely, through the Moodle platform, the Postgraduate Program in Science and Mathematics Teaching (PPGECIM), and a WhatsApp group;

Stage 6 — analysis of the data obtained through the questionnaires, written records, and activity files sent by students via Excel on the Moodle platform.

6 The teaching sequence

According to Pannuti (2004, p. 4), a didactic sequence “consists of a series of planned and guided actions aimed at promoting specific and defined learning”. In this sense, the didactic sequence developed for the research consists of a comic strip entitled *Aurora and Cryptography*, activities involving historical encryptions and ciphers with matrices, as well as videos demonstrating encoding and decoding activities.

The comic book follows Aurora (Figure 2), a student who is passionate about mathematics and who, during her vacation, dedicates herself to studying cryptography. Fascinated by her discovery of this subject, Aurora decides to share her learning with her friends when classes resume. She proposes that they solve encryptions, believing that this will help them learn more about the topic. From there, Aurora and her friends embark on an adventure of encoding and decoding using historical encryptions and matrix ciphers.

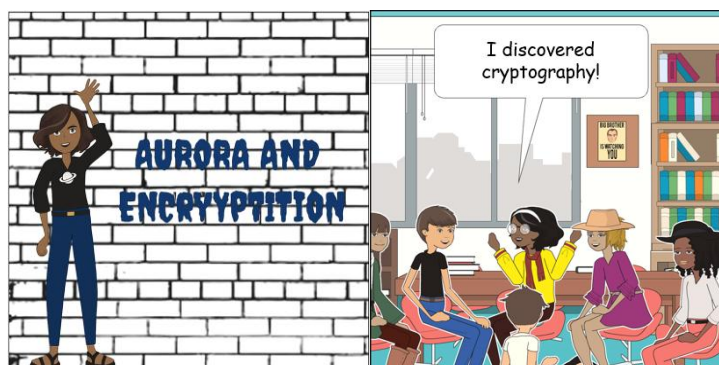


Figure 2: Comic strip from the comic book (Kranz, 2021, p. 74)

The activities in the teaching sequence covered the historical encryptions of Vigenère, Playfair, ADFGVX, and Hill, as well as the MKO encryption, developed as part of the research to explore operations with matrices. Thus, ten activities were developed for each of the encryptions, organized into five files, each containing two activities of each type. To familiarize students with Excel software, all activities developed for the teaching sequence were set up in spreadsheets. Table 2 presents a summary of the teaching activities proposed in the research.

Table 2: Summary of teaching activities in the teaching sequence

Teaching activity	Objective
Vigenère encryption	Understand and apply the procedures used to decode a message using the Vigenère encryption.
ADFGVX encryption	Understand and apply the procedures used to decode a message using the ADFGVX encryption.
Playfair encryption	Understand and apply the procedures used to decode a message using the Playfair encryption.
Hill encryption	Understand and apply the procedures used to decode a message using the Hill encryption.
MKO encryption — addition of matrices	Explore the operations of adding matrices and transposed matrices for encoding and decoding a message with the MKO encryption.
MKO encryption — subtraction of matrices	Explore the operations of matrix subtraction and transposed matrix for encoding and decoding a message with the MKO encryption.
MKO encryption — multiplication of matrices	Explore the operations of matrix multiplication and inverse matrix for encoding and decoding a message with the MKO encryption.
MKO encryption — matrix multiplication by scalar and matrix addition	Explore matrix addition, matrix multiplication by a scalar, and transposed matrix operations for encoding and decoding a message with the MKO encryption.
MKO encryption — matrix multiplication by scalar and matrix subtraction	Explore matrix subtraction, matrix multiplication by a scalar, and transposed matrix operations for encoding and decoding a message with the MKO encryption.

Source: Kranz (2021, p. 73)

Next, we present the activities developed based on the ADFGVX encryption and the MKO encryption, which address the historical aspect and explore the content of matrices, respectively.

The ADFGVX encryption is composed of a combination of substitution and transposition methods (Singh, 2003; Urgellés, 2018). First, each letter of the message is replaced by a pair of letters obtained from a 7×7 table containing the letters of the alphabet and numbers, by matching the row with the column. Next, the transposition method is applied, which requires the definition of a keyword without repeated letters. From this word, a table is created whose first line consists of the keyword and the following lines consist of the characters determined in the first part of the encryption. After this process, the keyword is organized in alphabetical order, thus reorganizing its columns. With the application of the two methods, the message is encrypted and can be forwarded from the lines of the table.


Figure 3 shows an activity from ADFGVX encryption that is part of the teaching sequence.

ADFGVX ENCRYPTION

I am sending you a secret message. This phrase was taken from a song that I really like. To find it, you will need to use the ADFGVX Encryption, using the keyword **MUSICA (MUSIC)**.

The message is:
“GDVXGD AGFVVV XAFVVV GDAVDD XXVXAX DGFDDA”

Use the ADFGVX Square to discover Aurora's secret message.



	A	D	F	G	V	X
A	X	K	Z	D	T	P
D	R	9	0	M	F	3
F	B	6	G	8	4	J
G	N	1	W	H	A	U
V	5	E	S	Y	O	Q
X	L	V	C	2	7	I

Figure 3: Activity of the Encryption ADFGVX (Kranz, 2021, p. 86)

To decode the message sent, follow the encoding steps in reverse order. This reorganizes the message according to the alphabetical order of the letters in the keyword. Next, the original layout of the columns is restored, reallocating the characters according to the keyword. Finally, the table is used to replace each digraph with the corresponding letter, revealing the original text (Figure 4).


SOLUTION											
STEP I						STEP II					
A	C	I	M	S	U	M	U	S	I	C	A
G	D	V	X	G	D	X	D	G	V	D	G
A	G	F	V	V	V	V	V	V	F	G	A
X	A	F	V	V	V	V	V	V	F	A	X
G	D	A	V	D	D	V	D	D	A	D	G
X	X	V	X	A	X	X	X	A	V	X	X
D	G	F	D	D	A	D	A	D	F	G	D
STEP III											
LET'S ALLOW OURSELVES 01											

Figure 4: Decoding the message with the ADFGVX encryption (Own elaboration)

The MKO Encryption was developed by us and explores concepts and operations with matrices, such as identity matrices, transposed matrices, inverse matrices, matrix addition, matrix subtraction, matrix multiplication by a scalar number, and matrix multiplication. To encode or decode messages with MKO Encryption, it is necessary to perform the indicated matrix operations, use the encoding/decoding alphabet, and the key matrix.

Figure 5 shows an activity from MKO Encryption that is part of the teaching sequence. The activities demonstrate the feasibility of developing a teaching sequence that relates the content of matrices to the topic of cryptography, integrating the use of Excel spreadsheets.

MKO Encryption



I am sending you a secret message. To discover it, you will need to use the MKO Encryption, following the steps indicated. The coded message is:

| 25 | 16 | 21 | 16 | 37 | 16 | 13 | 15 | 15 | 31 |

The key matrix for this message is:

2	7
9	11
7	8
4	1
2	3

The encoded message matrix has its elements distributed in columns. To decode this message, you must subtract the transpose of the key matrix from the message matrix. To discover the secret message sent, follow these steps:

FIND THE TRANSPOSED MATRIX

Start the transposed matrix of the key matrix at cell P7.

SHOW MESSAGE MATRIX

Start the transposed matrix of the key P16.

REVEAL THE ORIGINAL MATRIX

Start the original matrix at cell Y7.

WHAT IS THE MESSAGE?

Use the encoding/decoding alphabet on the side to discover the message.

**ALPHABET
ENCODER/DECODER**

A	B	C	D	E	F	G
5	4	7	6	9	8	11
H	I	J	K	L	M	N
10	13	12	15	14	17	16
O	P	Q	R	S	T	U
19	18	21	20	23	22	25
V	W	X	Y	Z	Ç	+
24	27	26	29	28	31	30

Figure 5: Activity of MKO Encryption (Kranz, 2021, p. 90)

7 Description and analysis of the teaching sequence

The research was developed with 23 students in the 3rd grade of High School at the São João Batista State Technical School, in Montenegro. Due to the Covid-19 pandemic, the activities were carried out remotely, using the PPGEICIM's Moodle virtual learning environment. Data production took place between September and October 2020, during six meetings that included the application of questionnaires, the comic strip developed for the research, and five activities that explored historical figures and the content of matrices.

To analyze student records, data sent to the Moodle platform was considered. However, due to the online format and the context of the pandemic, only six students completed all the proposed activities at the end of the meetings. Thus, the data analyzed refer to these participants, who were identified as A1, A2, A3, A4, A5, and A6.


The BNCC (Brasil, 2018) points out the importance of students valuing and using

historically constructed knowledge to understand and explain reality, in addition to continuing to learn. In this sense, the comic book *Aurora e a Criptografia* [Aurora and Cryptography] and the activities with historical encryptions aimed to demonstrate the relevance of this knowledge over the centuries and exemplify the evolution of cryptographic methods. To exemplify historical encryptions, the solution to an activity involving the ADFGVX encryption was presented.

Solving the ADFGVX Encryption activity required students to use the ADFGVX Encryption table to find the letter corresponding to the intersection of the digraph of the encoded message, after organizing the encoded message into lines according to the keyword. Thus, this table has an alphabet and numbers in random order, which refers to the idea of a 6×6 matrix. From this encryption, one can explore the ideas of arranging elements in rows and columns, as concepts of matrices.

To carry out the activity with the ADFGVX encryption, student A4 (Figure 6) used a grid sheet, which resembles the cells of a spreadsheet, to develop the proposal.

ADFGVX ENCRYPTION



I am sending you a secret message. This phrase was taken from a song that I really like. To discover it, you will have to use the ADFGVX Encryption, using the keyword MUSICA (music). The message is:

"FVVGVA G DVVGF VAAVGD VVDVVD FVVGVA AXVVVF FDAGDV"

Use the ADFGVX Encryption table to discover Aurora's secret message.

	A	D	F	G	V	X
A	X	K	Z	D	T	P
D	R	9	0	M	F	3
F	B	6	G	8	4	J
G	N	1	W	H	A	U
V	5	E	S	Y	O	Q
X	L	V	C	2	7	I

Activity 2:

ATIVIDADE 2:

ACIMSU	MUSICA	
FVVGVA	GAVVVF	GA VV VF
GDVVGF	VEGVGD	VF GV DG
VAAVGD	VDGAAV	VD GA AV
VVDVVD	VDDVVF	VD VD VV
FVVGVA	GAVVVF	GA VV VF
AXVVVF	VFVXYA	VF VV XA
FDAGDV	GVDADE	GV DA DF

→ Answer
Our brain is our home.


Figure 6: Solution to exercise from ADFGVX Encryption by student A4 (Kranz, 2021, p. 108)

Student A5 (Figure 7) solved the ADFGVX Encryption activity using an Excel spreadsheet.

It should be noted that different decoding activities were created for each group or student, allowing each one to work according to their preference.

As discussed in Olgin (2015), cryptography can serve as a resource for contextualizing mathematical content, such as matrices, enabling the development of activities that integrate matrix operations with encryptions and codes. In this sense, the activities developed based on MKO Encryption combined the study of matrices with cryptography, using Excel spreadsheets to perform matrix operations.

ADFGVX ENCRYPTION



I am sending you a secret message. This phrase was taken from a song that I really like. To discover it, you will have to use the ADFGVX Encryption, using the keyword MUSICA (music). The message is:

“XXDGVX DVVAGG VVVAGV ADVVGA DGVGDV”

Use the ADFGVX Encryption table to discover Aurora's secret message.

A	D	F	G	V	X
A	X	K	Z	D	T
D	R	9	0	M	F
F	B	6	G	8	4
G	N	1	W	H	A
V	S	E	S	Y	O
X	L	V	C	2	7

A	C	I	M	S	U
X	X	D	G	X	V
D	V	V	A	G	G
V	V	V	A	G	V
A	D	V	D	G	A
D	G	F	G	D	V

M	U	S	I	C	A
G	V	X	D	X	X
A	G	G	V	V	D
A	V	G	V	V	V
D	A	G	V	D	A
G	V	D	F	G	D


GV	XD	XX	AG	GV	VD	AV	GV	VV	DA	GV	DA	GV	DG	GD
A	V	I	D	A	E	T	A	O	R	A	R	A	0	1

Frase decodificada: 'A vida é tão rara' Answer: The life is so rare.

Figure 7: Solution to exercise ADFGVX Encryption by student A5 (Kranz, 2021, p. 108)

To develop the matrix multiplication activity with MKO Encryption, student A4 (Figure 8) correctly performed the inverse matrix calculations and matrix multiplication using the spreadsheet commands.

MKO ENCRYPTION



I am sending you a secret message. To discover it, you will have to use the MKO encryption, following the steps indicated. The encrypted message is:

| 5 | 30 | 15 | 5 | 16 | 51 | 17 | 13 | 10 | 31 | 11 | -5
| -30 | -14 | -5 | -13 | -72 | -17 | -17 | -4 | -40 | -13 |

The key matrix for this message is:

-1	2
2	-3

The coded message matrix has its elements distributed in columns. To decode this message, you must multiply the message matrix by the inverse of the key matrix.

To discover the secret message sent, follow these steps:

FIND THE INVERSE MATRIX

Start the inverse matrix of the key matrix at cell Q6

`=MATRIZ.INVERSO(I10:J11)`
`MATRIZ.INVERSO(matriz)`

SHOW THE MESSAGE MATRIX

Start the message matrix with cell M11

5	-5
30	-30
15	-14
5	-5
16	-13
51	-72
17	-17
13	-17
10	-4
31	-40
11	-13

REVEAL THE ORIGINAL MATRIX

Start the original matrix from cell Y6.

`=MATRIZ.MULT(M12:N22;Q6:R7)`
`MATRIZ.MULT(matriz1; matriz2)`

17	16
5	5
22	19
9	30
17	17
5	9
22	16
13	22
7	9

WHAT IS THE MESSAGE?

Use the encoding/decoding alphabet on the side to find out the message.

A Matemática nao mente
(The math doesn't lie)


Alphabet encoding/decoding

A	B	C	D	E	F	G
5	4	7	6	9	8	11
H	I	J	K	L	M	N
10	13	12	15	14	17	16
O	P	Q	R	S	T	U
19	18	21	20	23	22	25
V	W	X	Y	Z	C	*
24	27	26	29	28	31	30

Figure 8: Resolution of activity 2, from MKO Encryption, by student A4 (Kranz, 2021, p. 120)

The activity with the MKO Encryption of student A6 (Figure 9) involved the operation of subtracting matrices. It can be observed that A6 determined the transposition of the key matrix and the difference between the matrices using the commands of the spreadsheet.

MKO ENCRYPTION



I am sending you a secret message. To discover it, you will have to use the MKO encryption, following the steps indicated. The encrypted message is:

| 25 | 16 | 21 | 16 | 40 | 16 | 13 | 15 | 15 | 31 |

The key matrix for this message is:

2	7
9	11
7	8
4	1
2	3

The encrypted message matrix has its elements distributed in columns. To decode this message, you must subtract the transposed key matrix from the message matrix.

To discover the secret message sent, follow these steps:

FIND THE TRANSPOSED MATRIX

Start the transposed matrix of the key matrix at cell P7.

=TRANSPOR(W9:H9:I13)	4	2	
TRANSPOR(matriz)	8	1	3

SHOW THE MESSAGE MATRIX

Start the message matrix at cell P16.

25	21	40	13	15
16	16	16	15	31

FIND THE TRANSPOSED MATRIX

Start the transposed matrix of the key matrix at cell P7.

2	9	7	4	2
7	11	8	1	3

SHOW THE MESSAGE MATRIX

Start the message matrix at cell P16.

25	21	40	13	15
16	16	16	15	31

REVEAL THE ORIGINAL MATRIX

Start the original matrix at cell Y7.

=P16:T17-P7:T8	33	9	13		
	9	5	8	14	28

WHAT IS THE MESSAGE?

Use the encoding/decoding alphabet on the side to discover the message.

Seja feliz (Be happy)

Alphabet encoding/decoding

A	B	C	D	E	F	G
5	4	7	6	9	8	11
H	I	J	K	L	M	N
10	13	12	15	14	17	16
O	P	Q	R	S	T	U
19	18	21	20	23	22	25
V	W	X	Y	Z	C	*
24	27	26	29	28	31	30

Figure 9: Resolution of activity 1, from MKO Encryption, by student A6 (Kranz, 2021, p. 122)

It was observed that the MKO Encryption activities allowed students to manipulate Excel *software* commands to explore concepts and operations with matrices (Flores, 2021; Kranz, 2021), as well as the (re)organization of data when working with matrices.

Participants reported in the questionnaire sent on Moodle or WhatsApp that both the activities with historical figures and MKO Encryption and the theme of cryptography contributed to their understanding of the content of matrices, corroborating the proposal for mathematical contextualization based on themes (Brasil, 2006, 2018, 2019; Olgin, 2015; Rodrigues and Sá, 2019).

Furthermore, Student A1 highlighted the usefulness of spreadsheets: “*They provide us with an easier way to solve some of the situations in which we need to multiply or add matrices, for example.*” This highlights the role of digital technologies as facilitators of learning, allowing students to build knowledge and solve academic or social problems (Brasil, 2018).

In addition, Student A6 stated that the use of Excel helped them to better understand the content of matrices, realizing that these can be represented by tables and the operations performed more simply in the *software*. Thus, the topic of cryptography, associated with the security and privacy of digital information, and the use of spreadsheets provided a practical link to mathematical content (Olgin, 2015; Rodrigues and Sá, 2019).

Therefore, it is considered important to integrate technological resources and topics into classroom activities, seeking to contextualize mathematics teaching in high school (Olgin, 2015; Brasil, 2018). Thus, the activities in the teaching sequence represent teaching proposals for working with matrix content through the theme of cryptography combined with the use of Excel software.

8 Final thoughts

The research conducted is in line with studies in Mathematics Education that highlight the importance of a contextualized approach to teaching mathematical content. It is understood that developing content through themes can facilitate students' understanding, bringing them closer to relevant real-world issues and contributing to their civic education.

The results of applying the sequence indicate that the historical approach, integrated with the comic book *Aurora e a Criptografia* [Aurora and Cryptography] and activities involving historical encryptions and matrix operations, together with the use of digital technologies, provided reflective and meaningful teaching. The contextualization of matrix content using spreadsheets provided an opportunity to understand the content and to expose students to digital technologies and their potential.

Therefore, the research proposed a set of activities that enabled students to work on matrix content through the theme of cryptography. This encouraged them to become active participants in solving the problems, using Excel to rethink their strategies and establish connections between the theme and mathematical concepts.

However, the low participation rate due to the remote context was a limitation of the research, since the students had little interaction with each other and with the researcher. In asynchronous moments, each student performed the planned activities individually, which resulted in less exchange of ideas, making it difficult to clarify doubts and build knowledge collectively. Thus, for future applications of this research, a larger number of participants and a face-to-face context should be considered in order to make comparisons with the results obtained in this investigation.

It should be emphasized that, when proposing a didactic sequence that explores work with themes and mathematical concepts, it is essential to employ diversified strategies that help demonstrate the practical applicability of mathematics in students' daily lives. In addition, careful planning of activities and the selection of appropriate methodologies and resources are crucial to achieving the teaching objectives. Furthermore, it is also necessary to investigate topics related to mathematical content, with the intention of promoting teaching that is contextualized to the students' reality.

In this sense, the integration of different subjects allows students to perceive the applicability of mathematics in different aspects of everyday life. In addition, this approach favors the development of critical thinking, informed decision-making, and interdisciplinarity, promoting the articulation of mathematical concepts with real-world issues in the Mathematics Curriculum, which provides opportunities for dynamic, inclusive teaching that prepares students for the challenges of life in society.

Conflicts of Interest

The authors declare that there are no conflicts of interest that could influence the results of the research presented in the article.

Data Availability Statement

The data produced and analyzed in the article will be made available upon request to the authors.

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