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Educação Matemática Debate

Teacher professional knowledge, curriculum materials (educative) and the teaching of linear functions: some potentialities

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ARTICLE

Abstract: This study aimed to analyze the potential of an educative curriculum material for teaching linear functions, from the perspective of High School Mathematics teachers and the knowledge they mobilize when interacting with this material. To this end, we used qualitative research, developed from a semi-structured interview with two teachers. The results obtained from the analysis of the material, considering theoretical reflections on professional teaching knowledge and the teacher-curriculum material relationship, indicate that teachers identify the potential advantages of the material's implementation confidence, flexibility, investigative approach, and arousing student interest in the content.

Keywords: Educative Curriculum Material. Material Potential. Teacher Professional Knowledge. Linear Function.

Conocimiento profesional docente, materiales curriculares (educativos) y la enseñanza de la función afín: algunas potencialidades

Resumen: El estudio tuvo como objetivo analizar las potencialidades de un material curricular educativo para la enseñanza de la función afín, desde la perspectiva de profesores de Matemáticas de Secundaria y los conocimientos que movilizan al interactuar con este material. Para ello, adoptamos un enfoque de investigación cualitativa, desarrollando una entrevista semiestructurada con dos profesores. Los resultados del análisis del material, considerando las reflexiones teóricas sobre el conocimiento profesional docente y la relación Profesor-Material Curricular, indican que los profesores identifican como potencialidades la seguridad al implementar el material, su flexibilidad, la presencia del enfoque investigativo y el despertar del interés de los estudiantes por el contenido desarrollado.

Palabras clave: Material Curricular Educativo. Potencialidades del Material. Conocimiento Profesional Docente. Función Afín.

Conhecimento profissional docente, materiais curriculares (educativos) e o ensino de função afim: algumas potencialidades

Resumo: O estudo objetivou analisar as potencialidades de um material curricular educativo para o ensino de função afim, sob a perspectiva de professores de Matemática do Ensino Médio e os conhecimentos mobilizados por eles ao interagirem com este material. Para isso, utilizamos da pesquisa qualitativa, desenvolvida a partir de uma entrevista semiestruturada com dois professores. Os resultados alcançados com a análise do material, considerando as reflexões teóricas acerca do conhecimento profissional docente e da relação professor-material curricular, apontam que os professores identificam como potencialidades a segurança na hora de implementar o material, a flexibilidade, a existência da abordagem investigativa e o despertar do interesse dos estudantes quanto ao conteúdo elaborado.

Palavras-chave: Material Curricular Educativo. Potencialidades do Material. Conhecimento Profissional Docente. Função Afim.

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1 Introduction

Throughout their teaching practice, teachers are in constant contact with curriculum materials that help them improve their teaching and strengthen their performance. In Lima and Januario (2021), the term curriculum materials is used, within the scope of Mathematics Education, to refer to the resources used by teachers in the classroom when promoting and mediating learning situations. These materials can include textbooks, handouts, materials produced by non-governmental organizations, activity books developed by education departments, among others.

In line with this thought, in Lima, Januario and Perovano (2024) it is stated that, in Brazil, textbooks are the most widespread and used curriculum materials. Similarly, Prado (2014, p. 16) points out that "such curriculum materials are an active presence in Mathematics classrooms in Brazil". It is notorious that many teachers make use of curriculum materials, given their influence on teaching practice and the development of student learning. In this sense, *educative curriculum material* (ECM) can be defined as "those designed with the intention of promoting teacher learning, and not just those intended for student learning" (Lima, Januario and Perovano, 2024, p. 33). In this way, it is possible to further assist teaching practice, which can also reflect on student learning. It is important to note that this research will use an educative curricular material, prepared by one of the authors, for teaching linear functions using GeoGebra software. Based on the initial discussions held so far, the study[1] aimed to analyze the potential of an educative curriculum material for teaching related functions, from the perspective of High School Mathematics teachers and the knowledge mobilized by them when interacting with this material.

In order to meet this objective, we based ourselves on the ideas of Brown (2009) and Remillard (2009) on the teacher-curriculum materials relationship and on the ideas of teacher professional knowledge, defended by Ball and colleagues (Ball, Thames and Phelps, 2008; Ball and Bass). In addition, we present some ideas on the use of digital technologies and the GeoGebra software, as it is a subsidy for the educative curriculum material developed.

In addition to the Introduction, this paper is organized as follows: in section 2, we discuss teacher professional knowledge. In section 3, we discuss the relationship between teachers and curriculum materials. In section 4, we present the GeoGebra software and the teaching of functions. In section 5, we describe the context and characterization of the research. In section 6, we discuss the potentialities of ECM and teacher professional knowledge. Finally, in the last section, we present the final considerations of this work.

2 Professional teaching knowledge

During the production or application of different activities in the teaching context, teachers are mobilizing knowledge at all times (Bueno, Alencar and Oviedo, 2017; Barbosa and Lopes, 2020). In order to better understand the knowledge they access during the course of their profession, we have come across various theoretical models that deal with professional teaching knowledge. For this reason, taking into account the context of this research, which is within the scope of Mathematics Education, we will approach the theoretical model proposed by Ball, Thames and Phelps (2008), which presents specific categories for teachers who teach Mathematics.

These authors propose the notion of Mathematical Knowledge for Teaching, structuring it into two domains, divided into six subdomains, as represented in Figure 1. On the left side, the domain of *subject matter knowledge* is presented, and its three subdomains: *Common*

[1] This article is an excerpt from a monograph defended in the Mathematics Degree course at the Federal University of Recôncavo da Bahia, written by the first author and supervised by the second author.

Content Knowledge; Specialized Content Knowledge; and *Horizon Content Knowledge*.

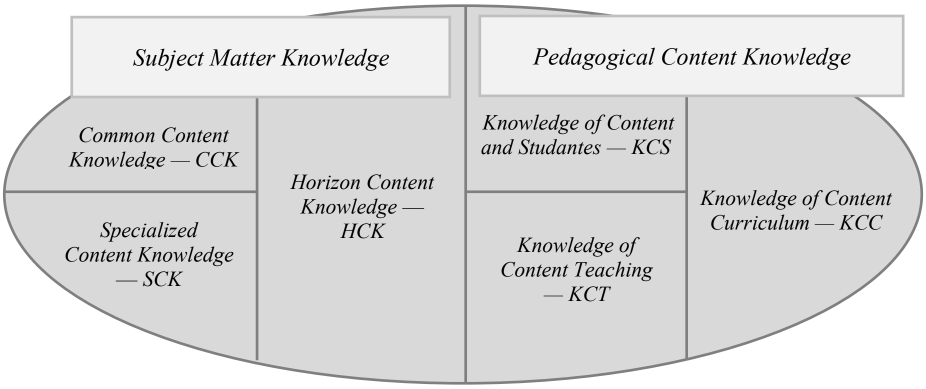

Figure 1: Domains of Mathematical Knowledge for Teaching - MKT (Ball, Thames e Phelps, 2008, p.403).

Common content knowledge (CCK) is "the knowledge used in the work of teaching in a common way as it is used in many other professions and occupations that also use Mathematics" (Hill, Ball and Schilling, 2008, p. 377). This is the mathematical knowledge mobilized to solve simple tasks, activities and mathematical problems. Because it has been formed over the years, it is considered common knowledge, built up through the teacher's experiences, but it is not exclusive to the teacher, as it is used in various contexts and professions. As an example of this, we can cite the fact that a salesman who needs to charge 50.00 reais on a 100.00 reais note knows that, in order to do so, he needs to return 50.00 reais. This shows that they mobilize their common knowledge to carry out this task.

As far as *specialized content knowledge* (SCK) is concerned, it is mobilized by teachers to represent situations mathematically, present mathematical strategies and procedures to students so that they can solve the proposed problems, validate these procedures and the answers obtained with mathematical arguments. Unlike common knowledge, this is only and exclusively used for teaching Mathematics (Ribeiro, 2016).

With regard to *horizon content knowledge* (HCK), it concerns "becoming aware (more like an experienced and thoughtful tourist than a tour guide) of the mathematical breadth in which experience and teaching are situated" (Ball and Bass, 2009, p. 6). This knowledge is related to the teacher's observation of the contents in a broad way, relating them within the curriculum and verifying that these are not isolated points, as they complement each other. They are used by the teacher to prepare a lesson or choose a subject to be covered. This choice implies an understanding that the contents covered in a given school year are interconnected with those covered in other years, thus providing a form of anticipation and preparation for subsequent contents.

In addition to the knowledge presented in Figure 1, on the right side is the domain of *pedagogical content knowledge*, which is also divided into three subdomains: *knowledge of content and students; knowledge of content and teaching;* and *knowledge of content and curriculum.*

Knowledge of content and students (KCS) is "content knowledge that is intertwined with how students think about, know or learn this specific content" (Hill, Ball and Schilling, 2008, p. 375). It is mobilized by the teacher when he/she understands that the students have prior knowledge of a certain lesson or content. Based on this, the teacher prepares their lesson and chooses the best way to approach the content, from which point to start, and what possible difficulties the students will face.

As for *knowledge of content teaching* (KCT), this is mobilized by the teacher by mixing their knowledge of the content with their knowledge of teaching. This is the knowledge used by the teacher when organizing the content to be covered, choosing examples and activities that best meet their objective, whether it is to deepen or fix the students' knowledge of the subject. It is on this basis that the teacher observes the advantages and disadvantages of each procedure and methodology, and makes his or her choices.

Finally, *knowledge of content curriculum* (KCC) refers to “knowledge of curriculum guidelines, orientations, purposes and motivations, curriculum materials and sequencing of subjects at different school levels” (Ribeiro, 2016, p. 54).

Based on this theoretical model, it is possible to understand some of the stages that teachers need to go through when teaching, as well as the knowledge they mobilize at each stage in order to teach mathematics effectively. These stages are also present when choosing or constructing teaching materials, the focus of this research. In this study, we also consider the importance of the teacher-curriculum material relationship.

3 The relationship between teachers and curriculum materials

In order to better understand the relationship between math teachers and educative curriculum materials, we need to go through some discussions about this link with curriculum materials. Authors such as Davis and Krajcik (2005), Remillard (2005), Brown (2009), Lima, Januario and Perovano (2024) discuss this topic, how teachers interact with these materials, what knowledge is learned, shaped and modified during this interaction and how this can influence student learning.

Brown (2009, p. 17), for example, believes that pedagogical practices “have come to rely on curriculum materials as necessary tools for conducting and reproducing curriculum conceptions, forms and practices”, and from this, the teacher will make their choices and determine how this material will influence their teaching process. The author presents a theoretical framework that shows aspects of curriculum materials and teachers' resources, as well as how they interact with each other to promote teaching. Brown (2009) therefore believes that the teacher-curriculum material relationship is influenced by both the curriculum resources themselves and the teacher's resources.

With this in mind, we can point to the curriculum materials and the resources they influence during interaction, such as: physical objects; procedures; domain representation. The *physical objects* represent “the material nature of the curriculum materials themselves, including accompanying supplies” Brown (2009, p. 26), that is, it concerns the material itself that the teacher has in hand and will use. For example, we can think of a curriculum material consisting of an activity on equations, in which the teacher takes a scale into the classroom to show the property of the equivalence of equations. All the material at hand, including the scale, is part of the physical object.

Procedures “include instructions, *procedures* and *scripts* that are intended for teacher and student approval” Brown (2009, p. 27). This resource is brought in at the point where the material conveys to the teacher recommended ways of applying it and how to conduct the lesson.

Domain representation “refers to the form and organization of the domain of concepts and their relationships by means such as diagrams, models, explanations, descriptions and analogies” Brown (2009, p. 27). This involves demonstration through the examples and explanations provided in the material, as well as the sequencing of concepts.

In addition to what is provided by the curriculum material, an important issue in this relationship is how the teacher interacts with these aspects, because for Brown (2009) the use of curriculum materials does not offer a guarantee of change in teaching, the materials are a support for the teacher, who in turn brings some resources to use them, which are: content knowledge, pedagogical content knowledge, objectives and beliefs. It is worth noting that these are the categories of knowledge presented by the author. To analyze the data from this research, we used the categories presented by Hill, Ball and Schilling (2008), as they present specificities of mathematics teaching that are important for this work.

Content knowledge deals with knowledge of concepts, ideas and facts within a given domain. It refers to the knowledge that the teacher already has about a particular content, and can be related to common content knowledge, described by Hill, Ball and Schilling (2008) as knowledge about mathematical aspects, but it is not exclusive to the teacher.

Pedagogical content knowledge “generally brings together pedagogical knowledge with domain knowledge to describe the knowledge of how to teach a particular domain” (Brown, 2009, p. 27). It corresponds to empirical knowledge, built up and studied in search of better content and teaching methods, taking into account the resources available. It is related to the *knowledge of content and teaching* described by Hill, Ball and Schilling (2008) in which knowledge about teaching and content are mixed.

Finally, *goals and beliefs* equate to “teachers' orientations towards the material they teach. This goes beyond their ability to teach something, to focus on their motivations for teaching it” (Brown, 2009, p. 27). This is directly linked to the teachers' personal aspects, what they like, what they accept, what they hold to be true and what drives them to teach the way they do.

These resources are interconnected in a bidirectional way by the types of appropriation that teachers can exercise over curriculum materials, which are: reproduction, adaptation and improvisation, that is, when using a material, depending on its resources, the teacher will reproduce it exactly as the authors of the material described. In addition, the teacher can adapt the material, making changes to what they deem necessary to suit their context, or improvise by creating spontaneous strategies during the lesson based on what is happening at that moment, and all of this occurs from the teacher's resources in interaction with the material's resources (Brown, 2009).

From this understanding of the two-way relationship between teachers and curricular Materials, we can see that it is not only the teacher who acts on the materials, but that the materials also influence the teacher's decision-making and can thus generate knowledge for them. Based on these discussions, studies have emerged on educative curriculum material, the main point of this research, a curriculum material aimed at teacher learning.

As shown above, the main difference between a curriculum material and an educative curriculum material is its focus, although both aim to promote teaching, but the former tends to be a tool for promoting student learning, this is its objective, designed and built for this. On the other hand, the second type focuses on teaching the two parties, teachers and students, with an emphasis on the educator's learning, making the educator not just a transmitter of knowledge, but also a being in constant change and learning.

According to Santana (2015, p. 15) “ECM aim to establish communication with teachers through supports developed with the aim of creating, for example, a particular mathematical content”. This communication between teacher and material is something very interesting to observe because, from this, the material can help the teacher in their practice, making them better prepared for what they may face, and can even promote the teacher leaving their comfort zone to test new methodologies. This support provided by educative curriculum materials is what we sought when we built an ECM, linked to the use of GeoGebra software, to teach linear functions. Because we used Geogebra to create the ECM, the next topic discusses the importance of this tool in teaching functions.

4 GeoGebra and function teaching

Traditionally, one of the first pieces of mathematical content covered in secondary school is functions. Its importance is clear due to its applicability in solving different situations in the social context, everyday life and other sciences. This idea is reinforced by the Parâmetros Curriculares Nacionais [National Curriculum Parameters]:

> The concept of function also plays an important role in describing and studying, by reading, interpreting and constructing graphs, the behavior of certain phenomena, both in everyday life and in other areas of knowledge, such as physics, geography or economics (Brasil, 2000, p. 43).

In addition, there is the statement that

> It is therefore up to the teaching of mathematics to ensure that the student acquires a certain flexibility to deal with the concept of function in different situations and, in this sense, through a variety of problem situations in mathematics and other areas, the student can be encouraged to seek a solution, adjusting their knowledge of functions to build a model for interpretation and investigation in Mathematics. (Brasil, 2000, p. 44)

In addition to the justification for teaching function content, the way in which it is taught should also be an important aspect to discuss, as this is a point defended in the Base Nacional Comum Curricular [National Common Curriculum Base — BNCC], which states as a mathematical competence for High School "to understand and use, with flexibility and precision, different mathematical representation registers (algebraic, geometric, statistical, computational, etc.), in the search for solutions and communication of problem results" (Brasil, 2018, p. 531). To help develop flexibility and precision in the use of mathematical representations, GeoGebra can be a viable tool for teaching this content.

GeoGebra is a mathematical, educational, free and easily accessible software that can even be used on cell phones (Android and IOS). It has a number of dynamic geometry features, among which we would highlight the most interesting for teaching functions: *i*) the ease with which graphs can be constructed, as all you have to do is type the law of the function into the software; *ii*) the possibility of performing calculations in real time, making it possible to change numbers in the laws of functions; *iii*) instant visualization of their effect on the graph; and *iv*) easy visualization of important points — maximum, minimum, points of intersection and roots — with a simple click.

In addition, what led us to link this software to the educative curriculum material we built was the autonomy provided to students during its use, as they can create, modify and manipulate the graphs in their own way, changing values in the function laws or moving specific points on the graphs. "The characteristics of GeoGebra enable the creation of scenarios for investigation, in which the student is able to experiment with situations in a dynamic process" Pereira (2012, p. 32). Thus, GeoGebra allows students to build models and test their hypotheses in it, allowing them to experiment and build knowledge in the process.

In addition, visualizing what happens to the graph with each change made to the law of the function — the opposite is also true — enables students to give visual meaning to their hypotheses. In this way, using GeoGebra can enable students to develop skills in working with function graphs and give more meaning to the content studied. Some of the skills relating to function content contained in the BNCC reinforce the idea that the use of GeoGebra can be beneficial to learning, as they suggest:

> (EM13MAT302) Build models using 1st or 2nd degree polynomial functions to solve problems in different contexts, with or without the support of digital technologies. (Brasil, 2018, p. 536)
>
> (EM13MAT401) Convert algebraic representations of 1st degree polynomial functions into geometric representations on the Cartesian plane, distinguishing the cases in which the behavior is proportional, whether or not using algebra and dynamic geometry software or applications. (Brasil, 2018, p. 539)

These two skills can be achieved through the use of GeoGebra, coupled with the teacher's guidance and the realization of guiding activities. It was with this in mind that we built an educative curriculum material which will be discussed in the next topic, as well as the methodology used in this research.

5 Context and characterization of the atudy

The study aimed to analyze the knowledge mobilized by High School Mathematics teachers when interacting with an educational curriculum material for teaching related functions, in addition to identifying the potential of the material as perceived by them. In order to achieve this objective, we used qualitative research, bearing in mind that "qualitative research is therefore concerned with aspects of reality that cannot be quantified, focusing on understanding and explaining the dynamics of social relations" Silveira and Córdova (2009, p. 32).

In order to achieve this goal, we decided to create an educative curriculum material on linear functions using GeoGebra software. We contacted two High School teachers to analyze the material. We then proposed an interview in order to observe both the knowledge they mobilized when analyzing the material and to identify the potential of the material as observed by these teachers. In this case, we used semi-structured interviews to collect data, which "are based on a basic outline, but not rigidly applied, allowing the interviewer to make the necessary adaptations" (Lüdke and André, 1986, p. 34). This research method allows us to get closer to the subjects of the research and favors the construction of unplanned conversations.

We therefore drew up a series of questions in order to maintain a constant and directed conversation with the interviewees, while leaving room for new questions to arise. It should be noted that "in the interview, the relationship that is created is one of interaction, with an atmosphere of reciprocal influence between the questioner and the answerer. Especially in interviews that are not totally structured [...]" (Ludke and André, 1986, p. 33), which is exactly what we tried to observe in the analysis carried out in this work.

In order to take part in this research, we sought out teachers who worked in secondary education and who had already taught linear function content. Of the teachers invited, only two accepted the invitation, and they were named Hellen and Lucas, fictitious names.

The teachers who took part in the study have a degree in mathematics from the Federal University of Recôncavo da Bahia. Hellen also has a degree in physics and a specialization in Financial Mathematics and the Science and Mathematics Teaching. She has worked for 9 years as a teacher at the Vale do Jiquiriçá Territorial Center for Professional Education (CETEP) in the city of Amargosa, Brazil, currently teaching 2nd and 3rd grade High School classes. In addition to his degree, Lucas also has a specialization in Science and Mathematics Teaching and has just completed the Master's in Mathematics from the Federal University of Recôncavo da Bahia. He is a teacher at Santa Bernadete State College, teaching 3rd grade High School classes. Both teachers have taught linear function content, which allows for a more in-depth reflection on the subject and the influence of this material in the classroom, due to their longer experience.

In order to start the research, we decided to build an educative curriculum material on linear functions linked to the use of GeoGebra, since we had already applied some activities on this content and obtained good results in the students' learning. The ECM is made up of four parts: the first contains the lesson plan along with the proposed sequence of activities; the second contains the resolutions of the proposed activities for the teacher; the third contains the didactic sequence with comments and suggestions on how the teacher could approach each question, with possible questions and directions to be taken when applying this sequence; and finally, the fourth presents comments on the possible answers, doubts and questions that the students may have during the application of the sequence.

The ECM was given to the invited teachers and, during the hand-over, we explained what an educative curriculum material was and showed them the specific content it covered. In addition, we explained what we expected from the analysis of the material, making notes about it, what potential could be highlighted, how it would be applied, what changes they would make, among other things. It was also explained that the interviews would be recorded and, for this, the teachers' authorization was requested beforehand. Once they had cleared up their doubts and agreed to take part, the dates for the interviews were set based on their availability. The interviews took place with each teacher individually and lasted around an hour each. They were conducted using the Google Meet platform, given its convenience and comfort for the interviewees.

The interviews began with questions divided into three blocks. The first sought to get the teachers to talk a little about themselves, their training, how they used to work in the classroom, their proximity to technologies and materials similar to those used by the interviewers. They were also asked to explain their methodologies and their greatest difficulties in the classroom. The second block was intended to get the interviewees to imagine themselves using this material and thus make critical observations, pointing out potentialities or difficulties in using it. They were also asked to answer whether they would use it and what reaction they expected from their classes when they used the material. The interviewees were free to give their opinions, whether positive or negative. Finally, in the third section, interviewees were again asked to imagine themselves implementing the material, but focusing on how they would do so during the introduction or review of the material, what changes they would make when implementing it, and how they would continue the work. This final section aimed to encourage the interviewees to present the professional teaching knowledge they use when teaching and analyzing teaching materials. For the purposes of this article, we present the analyses from section 2 of the interviews in the next section.

6 EMC potentialities and teacher professional knowledge

During the literature review, we discussed the relationship between teachers and curricular materials and the types of use teachers make of these materials. We took as our basis the studies presented by Brown (2009), as well as the theoretical model proposed by Ball, Thames and Phelps (2008) on teachers' professional knowledge. Considering these postulates, in this section we will present the research data and make the respective analyses of the knowledge mobilized by the participating teachers when interacting with an educative curriculum material for teaching linear functions, involving the use of GeoGebra and the potential for teaching perceived by them in this material.

We identified the potential of the educative curriculum materials that teachers noticed when analyzing these materials. Based on their perceptions, we analyzed the knowledge mobilized to perceive these potentialities. In this sense, Brown (2009) points out that, when using curriculum materials, teachers mobilize their knowledge to grasp the resources that these materials have to offer and, on the other hand, these materials offer supports that awaken teachers' knowledge. We consider these potentialities that the materials provide for teachers' actions and that they grasp to be affordances. Therefore, affordances are the possibilities that curriculum materials provide for their use. They are related to the meaning of the object, connecting perception to action and cognition, involving the adequacy of the interaction between the individual and the object or environment (Lima, 2017).

In order to begin the investigation into teachers' perceptions of EMC, we first looked to see if the teachers were aware of educative curriculum material:

> HELLEN: My first contact was in the pedagogical residency as a preceptor, it was you who introduced me to the material, in this case, the group you were part of and I also got to know other materials from the other residents who were part of the pedagogical residency, my first contact was this (sic).
>
> LUCAS: Look, I had never stopped to analyze Educational Curricular Materials, so, because of that, I had to find out what they were, and I saw from the outset that they are those that aim to educate both students and teachers, right?

Hellen had already learned about educative curriculum materials in the Pedagogical Residency program, in which she was a preceptor. So she already knew the main objectives of these materials and some of the formats that were presented to her during the course of the research. Professor Lucas, on the other hand, had never had contact with this type of material, but was keen to learn about it and did some research.

After gaining a better understanding of their knowledge of educative curriculum materials, we asked the interviewees to give their general thoughts on the material they had been given to analyze:

> HELLEN: When I read the material, I have an idea of the whole picture, of everything that happens in the teacher's role, from before I'm in the classroom to during it. With this material, I have a lot of interesting information, I have it already planned, I have the answers, I have what can happen in the classroom, what the students can ask, and the interventions I can make so that the activity runs as smoothly as possible, to meet those objectives, and this really gives the teacher more confidence to apply it in the classroom.
>
> LUCAS: In general, I thought it was a good piece of material, I believe that for this part that is proposed of understanding the general law of the linear function, knowing the impacts of the coefficients, understanding the issue of growth and decrease, I believe it manages to cover these ideas very well, the way you brought the *GeoGebra* approach into the didactic sequence added a lot in this sense of enabling the student to have a clearer understanding of all this, from what I also saw of the material, it allows some variations. For example, when the teacher has a lot of computer resources at their disposal, they can have the students handle each of these functions themselves, or when these resources are a little scarcer, the teacher can handle them themselves and the students can observe.

From Hellen's speech, it is possible to see that she perceives some potential in the material, such as the fact that it gives her greater confidence when applying the activity because it presents a lesson proposal in the general context of the situation, covering everything from lesson planning, the answers and questions that the students may have to the interventions that the teacher can make to solve the questions. The teacher highlights the fact that the material predicts what might happen in the classroom. However, it's worth pointing out that the information presented in the material are possibilities, and not all actions can be anticipated. It is possible that something new will happen.

Lucas, on the other hand, talks mainly about the didactic sequence in the material. He points out that the material covers all the content proposed and, therefore, for him, the use of GeoGebra can increase students' understanding, thus mobilizing *content and student knowledge*. This knowledge is also accessed by the teacher when they demonstrate knowledge about their students, including how the students will react to certain material (Hill, Ball and Schilling, 2008).

In addition, Lucas sees the potential of the material, considering it to be flexible to use because, depending on the amount of material available (computers), students can interact individually directly with the software or interact indirectly with the teacher as a mediator.

Based on these reflections, we observed that the reading done by each teacher had a different focus. Professor Hellen focused on the guidelines for the teacher, while Professor Lucas focused on the proposal of the tasks and the resources used for them.

In Lima (2017), it is discussed that resources, such as anticipation of students' responses, planning, difficulties and errors made by students, transparency in the material, among others, observed by both Hellen and Lucas, reflect discussions about the elements and characteristics of the materials that can enhance the teacher's learning.

After the general considerations about the ECM, the teachers were asked about the specific points of the ECM that caught their attention:

> HELLEN: I think the use of technology is important. In addition, the proposal of the activity is very interesting, bringing something investigative so that the student is the protagonist of his learning, this proposal I think was what caught my attention the most, I find this investigative nature very interesting.
>
> LUCAS: Look, in my opinion, what draws the most attention in this material is the use of GeoGebra, because for me what your sequence brings is something more, as I said, when this approach could be done without the use of GeoGebra, but the quality of the lesson, I believe, in the same amount of time, would drop a lot, because we wouldn't be able to make so many graphs in such a short time, as GeoGebra makes possible and also apart from precision, there are things you do in GeoGebra that you can't do on the board, so, in that sense, I believe that's what draws the most attention.

As seen in their answers, both of them cited the use of technology as a point that caught their attention, precisely because they were not used to using it, as they mentioned in the initial part of the interview. Teacher Hellen sees the activity as being of an investigative nature, thus making the students take on a leading role in the construction of their knowledge. From this, we can see that she mobilizes *knowledge of the content and the student* and *specialized knowledge of the content*, because when she perceives the potential of the activity as being of an investigative nature and interesting for teaching, she is mobilizing knowledge that is exclusive to the teacher. Mathematical investigation is a teaching methodology; however, by understanding that the activity can lead students to become the protagonists of their own learning, she is mobilizing knowledge about her students and how this activity can influence their learning Ball, Hill and Schilling (2008).

Lucas mentions other potentialities he saw in the activity, such as the possibility of interacting with a large number of graphs that would not be possible without the use of the software, as well as mentioning the precision achieved, which is difficult to achieve when drawing graphs by hand. This shows another type of knowledge mobilized by the teacher, *mediational*, which “concerns the knowledge that the teacher needs to have to use and evaluate technological materials and resources to enhance students' learning about a specific mathematical object" (Godino, 2009, p. 101). In this case, Lucas mobilizes his knowledge about the importance of using GeoGebra, drawing a parallel with what would not be possible without it. It's worth pointing out that the *mediational* knowledge mentioned is directly related to content and teaching knowledge, as it is also knowledge that the teacher has about the best ways to teach a particular content, but with the specificity of using technological resources to do so.

When expressing the benefits obtained from using the software, teacher Lucas was asked about the amount of time needed to apply an activity like this, as this could be one of the reasons that would make this approach difficult, but according to him:

> LUCAS: The material is already pre-analyzed, so we can just do a quick reading, make some adaptations, according to my school, according to my classes, but the material is good, because it helps and helps the teacher's work a lot, because it's already well analyzed, and this is wonderful for our busy life of several classes, several lessons.

When the teacher mentions that the material *"is already well analyzed"*, he is referring to the fact that the material contains information on various aspects of the lesson, such as the guidelines for the teacher and possible doubts from the students, because later in the interview, he states that

> LUCAS: It's a material that is not difficult to understand, quite the opposite, it already has tips on how the teacher can approach it, the possible doubts that the students may have, so all of this makes it easier for the teacher to use the material.

At this point, the ideas presented by teacher Lucas converge with what teacher Hellen had mentioned as most important in the material. Although, at the time of the first question, teacher Lucas had focused more on the content of the tasks, with the new question, he focuses on the teacher's guidance. He emphasizes the fact that the material helps the teacher in terms of approaches, possible doubts from the students and how to conduct the lesson.

Although we can understand that the materials can bring elements and guidelines to help the teacher conduct their lessons, we understand that the teacher is not a mere implementer. On the contrary, using the material is not simple. This work involves the teacher's knowledge of themselves and their students in order to shape the material to meet real classroom conditions (Lima, 2017).

In the analysis of the continuity of the interview, we see that teacher Lucas talks about some more points of the material that caught his attention:

> LUCAS: I think the material helps students understand the general law of linear functions and their graphical representation. When students start to insert the functions you've included in the material, it's quite easy for them to realize that the graph is a line, also in understanding the terms of the function and what impact they have on the behavior of the graph and also in understanding whether the function is increasing or decreasing. I believe that the material achieves these objectives well.

The teacher, from what he says, shows that the material has the potential to fulfill its objective of helping students learn functions. Furthermore, we can see the teacher's perception of his students' reaction: *"when the students start inserting those functions..."*. It's as if the teacher was in the classroom watching his class solving the activities proposed by the material. This is a clear demonstration of *knowledge of content and students* being mobilized, because, based on his experiences and relationships with his students, he is convinced of how his class will react and how this material will affect them.

After highlighting the points that caught their attention, the teachers were asked if they would use this material in the classroom. They said:

> HELLEN: Yes, I would use it, because of the characteristics of the material, which brings this investigative nature to the student, the student is discovering, right, that proposed knowledge, unlike the teacher coming in and saying "it's increasing when this happens, it's decreasing when this happens" so, there the student, he can see, he can visualize the function, he can discover, he becomes the protagonist because he's there investigating.
>
> LUCAS: I believe that depending on the moment, yes [...] I believe that the way you approached it is a positive way, because it brings technology with the aim of facilitating this teaching and learning process, it enriches the moment, so I believe I would use it, yes, without any problem.

As you can see, both teachers said they would use this material. It is also worth highlighting the fact that teacher Hellen once again mentioned her perception of the activity being investigative, making the student the protagonist of their learning. This point recurs in the teacher's speech, which made us realize how much the activity left a strong impression on her.

We noticed that, at this point, the teacher spoke in more depth and focused on the specific content of the material, comparing the ways of teaching functions, showing that teaching through an investigative activity is different from just explaining when a function is increasing or decreasing. We understand that this comparison is to show her view that the activity analyzed has more potential than a simply expository lesson, because, according to the teacher, the student can visualize and discover with this material, which reinforces the perception of these potentialities, in other words, the affordances. In this sense, the teacher mobilizes *knowledge of content and teaching*, since it “refers to the combination of knowledge of mathematical content with knowledge of how to teach that content” (Lima, 2017, p. 93). The teacher talked about ways of teaching specific content, directing her speech towards the growth and decrease of functions.

On the other hand, Lucas begins his speech by explaining that he would use the material, depending on the moment. He then pauses to think, probably recalling some aspects of the activity and reflecting on how he would use it in his classes. This moment of reflection is important for the teacher, as he makes a brief analysis of the possibility of using this activity, and then judges the use of the material as positive, showing that he agrees with the way the technology is implemented. The teacher notices how this enriches the moment, making visible the mobilization of *specialized knowledge of the content* when, based on his knowledge as a teacher, he understands the material as beneficial for teaching, showing that he would use it in his classes.

In order to further discuss the use of the material, we asked them the following question: how do they imagine their students would react if they used this material?

> HELLEN: I think they would like it, because of the technology, it attracts attention, right? “Oh, the teacher took us to the computer room” or “The teacher told us to download an app”, that's already curious, it's already different. In addition, because of this very question, of being investigative, the student starts to investigate the activity and this can also attract their attention, their interest, so I believe they would like it and participate for these reasons.
>
> LUCAS: A good number of them would like this material, a good number of them would also feel more interested in the class, because it's something they like, working with technology. And the material also allows them to make their own discoveries, so depending on how the teacher

> conducts it, which also counts for a lot, I believe it could be a moment when the students would be more interested than simply having a lecture without using GeoGebra.

The teachers' ideas converge again when they note that, due to the use of technology, their students would enjoy the activity and feel more interested, curious, and this would increase their participation. The act of interpreting the possibilities of their students' reactions is directly linked to the mobilization of *content and student knowledge*. This is very evident when teacher Hellen says: *"Oh, the teacher took us to the computer room", "The teacher told us to download an app"*. You can see that the teacher places herself in the school space, imagining how her students would react, just as teacher Lucas did earlier. In addition to these potentialities perceived by the teachers, they also highlight the fact that the material gives the students autonomy to construct knowledge. In this respect, teacher Lucas makes an important observation about the *"conduct of the teacher"*, pointing out that the use of the material alone does not guarantee good learning, as many factors are involved during a lesson. Thus, even if the teacher chooses to reproduce the material faithfully, the students' reaction may vary and the teacher will have to make decisions based on this situation. This means that, even though the ECM aims to collaborate with the teacher in order to provide tools that support teaching activity and learning, it is the teacher who has the crucial role in developing the actions (Brown, 2009; Remillard, 2009; Lima, 2017).

Given that both teachers agree that using this material is a possibility, we asked targeted questions in order to find out what would limit them in this task and we got the following answers:

> HELLEN: Today at school we don't have that limitation, with the issue of computers, cell phones, they're no longer an obstacle as they used to be, I've stopped using activities like this because of that, now we have several *Chromebooks*. I don't know if you can download it, but, if I'm not mistaken, you can do the activity online, right? You can do it online, or on your cell phone, which most students have, we have internet, so I don't think there are any limitations.
>
> LUCAS: I think it's really up to the teacher whether they want to use it or not. They may not have 30 computers available, or at least 20, to put the students in pairs, but in my opinion, the didactic sequence allows you, with a single computer and a projector, to do the didactic sequence with the necessary adaptations, it's very much up to the teacher whether they want to use it or not.

The teachers interviewed say that they don't see any limitations in applying this activity, and both talk about technological resources. Teacher Hellen explains that, some time ago, this could have been an obstacle, but that the school now has Chromebooks and internet, and that students generally have access to cell phones, making it easier to use this material.

Teacher Lucas also points out that, even with few technological resources, only a computer and a projector would be needed, with *"the necessary adaptations"* being made so that this activity could be applied, so that the only limitation is the teacher's willingness to use this material.

From this, we conclude that one of the potentialities that these teachers saw in the material, mentioned earlier, was the fact that it was flexible to use, and could be applied via computers or cell phones, even online, making it viable for use in different scenarios. On the other hand, Lucas makes an important point in his speech: he raises the issue of whether or not the teacher is willing to carry out a task of this kind. This has to do with the resources that the teacher brings into play when interacting with the curriculum materials which, according to Brown (2009), are content knowledge, pedagogical content knowledge and their beliefs and conceptions. In this case, for the teacher to agree to carry out a task like the one proposed, among other things, their conceptions about teaching and learning mathematics need to converge with what is proposed by the material.

Still on the subject of limitations, Lucas ponders on one aspect that can limit the use of the material, but which is not a limitation of the material itself, but rather the training of some teachers:

> LUCAS: From what I see today, this has a lot to do with our initial training courses. I see that there has been an evolution in the curriculum of these courses, and today most of them include some subject that involves the use of technology. CFP[2], for example, we've worked with *GeoGebra*, we've worked with calculators in teaching mathematics, so this kind of approach in undergraduate courses helps teachers feel more comfortable using technology in the classroom. I don't think *GeoGebra* is difficult to use, the material you've produced contains some information on how to use *GeoGebra* to do the activity, but whether you like it or not, it's something that ends up happening, this resistance when you don't know it.

The teacher says that he doesn't find the *GeoGebra software* difficult to handle and also highlights the fact that the material analyzed provides information on its use, but even so some teachers may be afraid to use it, because according to him one of the reasons for this is the lack of contact and knowledge about this type of technology. This means that teachers don't feel comfortable using materials of this nature. Professor Lucas attributes part of the reason for this attitude to the curriculum of the degree course, although he says that it has evolved nowadays, referring to his own training.

During his speech, the teacher shows that he has knowledge about the curriculum, presenting the mobilization of *content and curriculum knowledge*, which according to Ribeiro (2016, p. 54), refers to “knowledge of curriculum guidelines”, exactly what was shown by teacher Lucas.

With this, we were able to see the knowledge that the teachers mobilized when analysing the educative curriculum material, as well as their intended use and perceptions of the potential that this material offers.

7 Final considerations

Based on this research, we presented the concepts of curriculum materials and educative curriculum materials, as well as discussing the relationship between these and teachers who teach Mathematics from the perspective of Brown (2009). This revealed the importance of two important aspects to be discussed in this relationship: the resources made available by the curriculum materials and the way they are perceived by teachers from their own resources. Furthermore, given this interaction, teachers in turn perceive potential in the material, which Brown (2009) characterizes as affordances. On the other hand, Hill, Ball and Schilling (2008) present and characterize teachers' professional knowledge, which is used in this research to analyses teachers' interaction with the curriculum materials.

In view of this, we observed that the potential perceived by the teachers was directed at three specific aspects: the educative curriculum material as a whole, the activities proposed in isolation and the use of technology.

With regard to the educative curriculum material as a whole, the affordances perceived were: security when applying the material and help at this point, provided by the predictions of what might happen and what can be expected. We also found that the tasks proposed in the

[2] Teacher Education Center (CFP) of the Federal University of Recôncavo da Bahia (UFRB).

material take into account what is proposed in the guidelines suggested to the teacher.

According to the teachers interviewed, the tasks proposed in the ECM are flexible to use, with the possibility of being used in different ways and with different amounts of resources, with no obvious limitations. They also cover the content in a way that is appropriate for teaching linear functions, implement what is proposed in the school curriculum and the BNCC and also have a level of difficulty that is appropriate for the year of schooling. The teachers indicated that the material is interesting for teaching, as it is investigative in nature, giving the student autonomy, placing them in the role of protagonist in the construction of their learning and, finally, opening up space for the teacher to continue addressing the content of proportion and other functions.

With regard to the use of technology, the teachers pointed out the potential that caught their attention, such as increased student understanding and participation, as well as better visualization and precision of the formations, enabling a large number of constructions in a short period of time. In addition, with regard to the use of technology, the teachers emphasized its relevance, given that it was a way of preparing the students for other times when they could use technological resources again without causing strangeness in the students.

In addition to the perception of the material's potential, while the teachers were discussing it, it was possible to identify all the categories of teachers' professional knowledge: *common knowledge of content; specialized knowledge of content; horizontal knowledge of content; knowledge of content and students; knowledge of content and teaching; knowledge of content and curriculum.* This knowledge is directly linked to their experiences and their knowledge of their students, teaching and learning, given that each teacher's perception is unique.

Mobilizing knowledge to identify the potential of a material can determine the use intentions that the teacher will make of it. We use the term intentions of use because the data was collected through interviews and not by empirically observing the teacher's actions in the classroom.

The study presented in this article allows us to reflect on different aspects of teaching, opening up countless possibilities for future research. One of these possibilities is the application of this material in the classroom and a comparison with what was expected of teachers and what actually happens in the classroom, comparing the potential and the knowledge mobilized, also opening up space for an analysis of the types of use that teachers make of the curriculum material.

During the analysis, there was a discussion about whether the activity should be used as an introduction or not. In this respect, the teachers highlighted their points of choice, which makes us think of the possibility of further research into the loss or gain of potential, determined by the type of use of the same activity.

Another point opened up by the discussions was that some teachers avoid using technology. One of the reasons for this could be that they had little or no interaction with it during their training, which opens up the possibility of research into teacher training. In this respect, we are interested in knowing how the initial training of teachers who have had contact with these resources influences their use in the classroom. With this in mind, we see educative curriculum materials as potential resources for teacher learning and for planning and developing their lessons, but further research such as that mentioned in this paper and other studies on the subject could be developed.

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Conflicts of Interest

The authors declare that there are no conflicts of interest that could influence the results of the research presented in the article.

Data Availability Statement

The data produced and analyzed in the article will be made available upon request to the authors.

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