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Educação Matemática Debate

A comparative study between the performance of 4th and 6th grade students in solving Sequence of Patterns tasks

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Article

Abstract: This paper aims to investigate and compare the performance of 4th and 6th grade students in solving tasks involving Sequence Patterns of the Crescent and Repetitive types, in the iconic and numerical contexts, using qualitative and quantitative methodological approaches. The study involved 81 students who solved four sequence pattern tasks. The analysis was based on relevant theories and test statistics. The results showed that 6th graders got 68% right, while 4th graders got 47%. Despite this difference, the statistical analysis did not consider it significant. This study highlights the relevance of research to understanding algebra performance in the early years of primary school, suggesting new areas for investigation.

Keywords: Early Algebra. Sequence Pattern. Early Years. Final Years.

Un estudio comparativo entre el desempeño de estudiantes de tareas de Secuencia de Patrones

Resumen: Este artículo tiene como objetivo investigar y comparar el desempeño de los estudiantes de 4° y 6° grado en la resolución de tareas que involucran Patrones de Secuencia de los tipos Ascendente y Repetitivo, en los contextos icónico y numérico, utilizando enfoques metodológicos cualitativos y cuantitativos. El estudio involucró a 81 estudiantes que resolvieron cuatro tareas de patrones en secuencias. El análisis se basó en teorías relevantes y estadísticas de prueba. Los resultados mostraron que los estudiantes de 6º grado tuvieron un 68% de respuestas correctas, mientras que los estudiantes de 4º grado lograron un 47%. A pesar de esta diferencia, el análisis estadístico no lo consideró significativo. Este estudio destaca la relevancia de la investigación para comprender el rendimiento del álgebra en los primeros años de EF, sugiriendo nuevas áreas de investigación.

Palabras clave: Álgebra Temprana. Patrón Secuencial. Primero Anos. Ultimos Anõs.

Um estudo comparativo entre o desempenho de estudantes dos 4º e 6º anos na resolução de tarefas de Sequência de Padrões

Resumo*:* Este artigo tem como objetivo investigar e comparar o desempenho de estudantes dos 4º e 6º anos na resolução de tarefas que envolvem os Padrões de Sequências dos tipos Crescente e Repetitivo, nos contextos icônico e numérico, utilizando abordagens metodológicas qualitativas e quantitativas. O estudo envolveu 81 estudantes que resolveram quatro tarefas de padrões em sequências. A análise baseou-se em teorias relevantes e estatísticas de teste. Os resultados mostraram que os estudantes do 6º ano tiveram 68% de acertos, enquanto os do 4º ano alcançaram 47%. Apesar dessa diferença, a análise estatística não a considerou significativa. Este estudo realça a relevância da pesquisa para entender o desempenho em Álgebra nos anos iniciais do EF, sugerindo novas áreas de investigação.

Palavras-chave: Early Álgebra. Padrão em Sequência. Anos Iniciais. Anos Finais.

1 Introduction

In Brazil, research into the teaching and learning of mathematics has been carried out in various fields since the middle of the 20th century, as Fiorentini (1994) states. One of the

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growing fields of research is the teaching of algebra in the first years of schooling, especially in the initial years of elementary school and in the first two years of the final years of the same stage of education. This interest has grown both nationally and internationally. Booth (1997), Blanton and Kaput (2005), Yamanaka and Magina (2008), Blanton *et al.* (2015), Ribeiro and Cury (2015), Jerônimo (2019), Porto (2018), Teixeira, Magina and Merlini (2021), among others, have contributed to the increase in research in this area.

In this sense, Bastos and Merlini (2020) consider that this research was one of the major drivers for the implementation of substantial changes in the school curriculum, especially in the Brazilian context. Within this context, the main contribution was the approval of the Base Nacional Comum Curricular [National Common Curricular Base — BNCC] (Brazil, 2017), the teaching of Algebra in Basic Education acquired greater importance. Thus, the document suggests that the teaching of initial ideas of algebra should be addressed from the early years of elementary school. However, it is important to note that the document guides the introduction of algebra without the use of letters or unknowns, thus focusing on intuitive concepts of regularity, patterns, generalizations and functional relationships.

The aim of teaching algebra in the early years[1] is to promote the development of algebraic thinking in children before algebraic concepts are formally introduced. In this way, the thematic axis should be developed through the understanding of patterns, equivalence situations, symbol manipulations and the modelling of abstract structures, such as generalizations of the pattern in sequence. This makes it possible to prepare students for the formal study of algebra, leading them to develop abstract skills and the use of arguments and generalizations (Dias and Noguti, 2023; Magina and Molina, 2023).

Among the contexts presented, this paper aims to present some results of an ongoing comparative diagnostic study carried out in the Graduate Program of a public university. The guiding question of this study is what are the strategies and performance presented by 4th and 6th grade elementary school students when solving activities involving the concepts of Sequence in Pattern? The aim is to investigate and compare the performance and strategies of 4th and 6th grade students in solving tasks involving Sequence Patterns of the Crescent and Repetitive types, in the iconic and numerical contexts.

This paper is organized as follows: first, we will discuss Algebra in the Early Years: theoretical perspectives, with the aim of providing a brief summary of the emergence and foundation of the title. We will then present the methodology used to collect and analyze the data. The results will then be discussed and compared, followed by our conclusions, which aim to present the main findings and implications.

2 The algebra in the Early Years: theoretical perspectives

Algebra is one of the main themes covered in mathematics, often presenting challenges in its definition. To overcome this difficulty, we turn to the definition proposed by Ponte, Branco and Matos (2009, p. 8). Algebra can be conceptualized as “a set of rules for transforming expressions (monomials, polynomials, algebraic fractions, expressions with radicals, etc.)”.

In the educational context, the BNCC defines algebra as a mathematical structure that uses letters and symbols and whose main purpose is to develop algebraic reasoning in students. In other words, the subject aims to help and motivate students to build mathematical models in order to understand, represent and analyze the quantitative and qualitative relationships between quantities. During the development of algebraic thinking, students need to be

[1] This approach includes teaching algebra to students from the 1st to the 6th grade of elementary school, since, according to the National Common Curricular Base (BNCC), the formal teaching of algebra takes place between the end of the 7th grade and the beginning of the 8th grade of Middle School.

encouraged to develop mathematical laws that represent the relationships of dependence between different variables.

But what actually is algebraic thinking? There are several approaches in the literature (Radford, 2014; Blanton and Kaput, 2005; Vale and Barbosa, 2019), however we will assume here the definition of Blanton and Kaput (2005, p. 413) who conceptualize algebraic thinking as "a process in which students generalize mathematical ideas from a particular set of examples, establish generalizations through argumentative discourse".

Algebraic thinking can take many forms, including functional reasoning, which is one of the main axes of this type of thinking. In addition, algebraic thinking can be seen as a process of constructing generalizations of patterns and relationships, using various linguistic and representational representations (Vale and Barbosa, 2019). It is important to note that this article focuses mainly on the development of students' functional reasoning, with an emphasis on the generalization of patterns and regularities.

For Van de Walle (2009, p. 296) "learning to look for patterns and how to describe, translate and extend them is part of doing mathematics and thinking algebraically". The ability to identify, analyze and extend patterns is an essential component of functional reasoning, as well as being a very important mathematical skill. According to Vale and Barbosa (2019), exploring patterns is a privileged way to introduce algebra due to the possibility of dynamically representing the variables involved.

According to Ponte, Branco and Matos (2009), the search for patterns and regularities, as well as the ability to formulate generalizations in a variety of contexts, should be encouraged from the beginning of primary education. These patterns, as pointed out by the authors, can be categorized as numerical patterns, geometric patterns, increasing patterns and repetitive patterns. For the purposes of this paper, we are focusing exclusively on increasing and repeating patterns.

The repetitive pattern, as defined by Vale *et al.* (2011, p. 20), is characterized as "a pattern in which there is an identifiable motif that repeats itself cyclically indefinitely". These repetitive patterns can be made up of icons or numbers that are constantly repeated.

According to Vale *et al.* (2011), in the early years, repetition patterns can be approached with different levels of exploration and connected to other mathematical content that is part of the annual curriculum. The authors emphasize the importance of providing students with problems that allow them to recognize the repeating pattern, describe, complete, continue and create the patterns. In this way, they are encouraged to explore diverse contexts, which in turn enables them to verbalize their thoughts based on their own justifications.

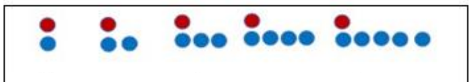
With regard to increasing sequences, each term in the sequence depends on the previous term and its position in the sequence, which is referred to as the order of the term (Ponte, Branco and Matos, 2009). According to the BNCC, the main skills for patterns in increasing sequences are recognizing and describing a pattern, completing missing elements in sequences and determining next elements in sequences. For Vale and Barbosa (2019), this type of task helps students in the early years to formulate conjectures and express the generalization that arises from inductive reasoning. They also encourage the formation of connections between different forms of representation. Conjecture, generalization and argumentation are more effective with this articulation, which improves understanding of the structure of basic mathematics. Given all the contexts presented, in the next section we will discuss the methodology of this study, which aims to investigate and compare the performance of 4th and 6th grade students in solving tasks involving increasing and repeating Sequence Patterns, in iconic and numerical contexts.

3 Methodology

In the research presented in this paper, we adopted a methodological approach that combines qualitative and quantitative elements. According to Flick (2009), the combination of these two approaches in analyzing the results can be considered complementary, providing a mutuality that enriches both the analysis and the final discussions. From this perspective, we used descriptive diagnostic research. Our aim with this type of research is to understand and compare the students' level of understanding and the signs of Functional Reasoning, based on the use of a diagnostic instrument. To carry out this study, an instrument was applied to a total of 85 students, divided into two different classes: 40 students from the 4th year and 45 from the 6th year of elementary school. The choice of these classes was based on specific criteria: i - students at different stages of schooling, covering both the initial and final years of elementary school; ii - based on the mathematical object, since both classes have already been taught multiplicative structures, but have not yet had direct contact with the concepts of Elementary Algebra, such as Equation, Inequation, Variable and Function, which will only happen from the 7th year onwards.

The diagnostic tool applied to the students was printed on an A4 sheet of paper, in notebook format, and consisted of nine different problems. Four of them are related to identifying the Functional Relationship[2], while the remaining five deal with Sequence Patterns (repetitive and increasing). Of these, two are repetitive (P1 and P6) and three are increasing (P5 and P7), both in iconic and numerical contexts. However, in order to meet the objective of the study, we only analyzed four of the five Sequence Pattern tasks. The problems analyzed are shown below in Table 1.

Table 1: Problems with the diagnostic tool used in this study

Iconic Repetitive Sequence Pattern	Numerical Repeating Sequence Pattern
P1. Look at the sequence of Emojis below:  a) Following this sequence, what is the next Emoji, that is, the Emoji in position 11? Draw the Emoji in position 11 in the space below. b) If I continued drawing the Emoji in this sequence, what would be the face in position 29?	P6. Ana loves to play sequences. Take a look at the sequence she made yesterday using the numbers 2 and 4. 2 | 4 | 2 | 4 | 2 1 st | 2 st | 3 st | 4 st | 5 st a) Following the same sequence, write down the next number in the sequence. b) Still following the same sequence as Anne, what number will occupy position 10 in this sequence?
Numerical Crescent Sequence Pattern	**Iconic Crescent Sequence Pattern**
P5. Look at the sequence of numbers below: 4 (position 1) 5 (position 2) 6 (position 3) ... (position 4) a) Following the same sequence, write down what will be the next number in this sequence? b) Still following the same sequence, what number will occupy position 10 of this sequence?	P7. Look at the sequence of figures made up of dots.  a) Following the same order, draw and paint the dots that will be needed to make the 6th position? b) Following the same sequence as the dots, how many dots will the 12th position have?

Source: Research data (2023)

The four problems listed in Table 1 present patterns in repetitive and increasing sequences, both in iconic and numerical contexts. The aim of these problems is to assess students' ability to understand, identify regularity and generalize sequence patterns. In items a, students must check the next term in the sequence, while in items b, the challenge is to observe

[2] This being another type of algebraic aspect

the term furthest from the last position, a generalization. Thus, this process requires a deeper understanding of the pattern, including consideration of the periodicity of the sequence and the ability to generalize.

The instrument was applied by one of the researchers, with the collaboration of the math teacher, in 4th and 6th grade classes, on different days, depending on the school's availability. The application was collective, but the solutions were individual, with no influence from the researcher or teacher. The application time was four hours for 4th grade and two hours for 6th grade. Each problem was read aloud to ensure comprehension, but no mathematical interpretation was carried out on the spot.

Data analysis was based on the qualitative interpretation proposed by Ludke and André (1986). It followed three stages: organizing the data into categories using a spreadsheet, comparative analysis of these categories in the light of theoretical references and critical review to identify elements that require further study.

In the study, the analysis encompasses both qualitative and quantitative aspects, incorporating a statistical evaluation using the *Chi-square*[3] *test and t-Students*, for comparative analysis based on the percentage rate of correct answers. The integration of these approaches enables a more comprehensive and in-depth understanding of the data. Thus, after all these methodological clarifications, in the next section we will present the results obtained from the application of the diagnostic instrument.

4 Results and discussions

We focused our analysis on the responses of 4th and 6th grade students, comparing their performance in solving tasks involving patterns of sequences, both in increasing and repetitive contexts. Once we had the data, we carried out two different approaches: a quantitative one, evaluating the students' performance, totaling 680 items[4], and a qualitative one. The quantitative analysis was divided into two main aspects: evaluation of performance within each group (4th and 6th grade), and comparison of performance between groups.

4.1 Intra-group performance

Table 2 shows the performance of 4th grade students in solving pattern tasks in repetitive and increasing sequences, in iconic and numerical contexts. Each problem consists of items *a* and *b*.

Table 2: Performance of 4th grade students in pattern sequence problems

<table>
<tr><th colspan="10">Pattern Sequence (Success = 58 %)</th></tr>
<tr><td colspan="4">Repetitive (Success = 62%)</td><td colspan="4">Growing (Success = 53%)</td><td rowspan="5">% Média</td><td rowspan="2">Numeric (Success = 50%)</td></tr>
<tr><td colspan="2">Numerical</td><td colspan="2">Iconic</td><td colspan="2">Numerical</td><td colspan="2">Iconic</td></tr>
<tr><td colspan="2">P 6 (Success = 61%)</td><td colspan="2">P1 (Success = 73%)</td><td colspan="2">P5 (Success = 36%)</td><td colspan="2">P7 (Success = 69%)</td><td>Iconic (Success = 71%)</td></tr>
<tr><td>A</td><td>B</td><td>A</td><td>B</td><td>A</td><td>B</td><td>A</td><td>B</td><td>Item A (Success = 71%)</td></tr>
<tr><td>72,5</td><td>50</td><td>75</td><td>70</td><td>62</td><td>10</td><td>72,5</td><td>65</td><td>Item B (Success = 49%)</td></tr>
</table>

[3] For Bassetto (2021), the Chi-Square test is considered a comprehensive, advantageous and effective method for analyzing various qualitative data involving two or more categories. It can be used through three different tests, i.e. adherence or adjustment, comparison and association.
[4] The result of multiplying the number of students by the number of items, i.e. eighty-five students multiplied by eight items.

Source: Research data (2023)

The analysis of the data in Table 2 shows that the 4th grade students got an average of 58% of the diagnostics right, with P1 standing out (73% correct), an iconic sequence with a repetitive pattern. In contrast, the numerical sequence with an increasing pattern was the most difficult for these students (36% correct).

When comparing the types of sequence, the results showed a percentage difference of 11% between repetitive and increasing, in favor of the former. This difference in performance was statistically significant according to the *Chi-Square test*[5] ($X^2 = 7.81$; $p = 0.005$), confirming that the students did better with repetitive sequences, i.e. 4th grade students showed greater aptitude in identifying patterns in repetitive sequences compared to increasing sequences. This result was also confirmed in the study by Vale *et al.* (2011), who pointed out that repetitive sequences are easier for students to identify than increasing sequences.

When we analyze the contexts of iconic and numerical sequence representations, we see a considerable disparity in the percentage of correct answers (22%), in favor of the iconic context. This difference between the percentage of correct answers was statistically significant, according to the *Chi-Square test* ($X^2 = 7.81$; $p = 0.005$). These results are in line with the findings of Jerônimo (2019) and Porto (2018). In fact, these authors concluded that the icons were a facilitating tool for students when solving problems.

In addition, the analysis of performance in relation to items A and B again revealed a marked difference (22% in favor of item A to the detriment of B). We used the Chi-Square test ($X2 = 7.380$ $p = 0.005$) and confirmed that this difference was not random. Remember that items A the value/position always asked for a position close to the last element/value in the sequence, while item B, where performance was much lower, asked for the identification of an element related to a more distant position, requiring students to generalize the sequence to some degree. This finding is close to the arguments put forward by Vale *et al.* (2011), for students to locate more distant terms in a sequence becomes more challenging as they move away from the terms initially presented to them. In this way, they indicate that longer sequences require students to show more elaborate reasoning.

In this sense, we focused on the results obtained by the 6th grade students. Thus, from the analysis of the results, there was a marked differentiation in their performance when approaching problems related to patterns in sequences, data presented in Table 3.

Table 3: Performance of 6th grade students in pattern sequence problems

Pattern Sequence (Success= 64%)									
Repetitive (Success= 66%)				Growing (Success= 61%)				**% Média**	Numeric (Success= 64%)
Numerical		Iconic		Numerical		Iconic			
P 6 (Success = 69%)		P1 (Success = 63%)		P5 (Success = 58%)		P7 (Success = 63%)			Iconic (Success = 63%)
A	B	A	B	A	B	A	B		Item A (Success = 81%)
80	58	87	38	87	29	67	58		Item B (Success = 46%)

Source: Research data (2023)

According to the results, the 6th graders showed a remarkable ability with patterns, achieving an overall average hit rate of 64%, with hits in each of the sequences varying by no

[5] These data were analyzed using a *p*-value significance level of less than 0.05.

more than 6% in relation to the average.

At first glance, this result indicates that the majority of these students have no difficulties in dealing with pattern sequences, whether repetitive or increasing, or within numerical or iconic contexts. This result was confirmed by the *Chi-Square test*, which showed that there was a significant difference between performance on repetitive and increasing problems, as well as between iconic and numerical problems.

Although the difference was not significant, we observed a slight advantage in student performance in repetitive sequences, to the detriment of increasing sequences. Vale *et al.* (2011) state that increasing sequences tend to complicate students' recognition of patterns, thus making it more difficult to recognize more distant terms.

However, when we analyzed items A and B, we noticed a percentage difference of 35% between the performance of item A and B, in favor of item A. This result was confirmed by the *Chi-Square test* ($X^2 = 75.094$ $p = 0.000$). Thus, we can assume that these 6th grade students manage to understand the initial terms of the sequence, but when they are faced with the need to find a term that is very distant from the last one represented, this proves to be an additional challenge for the student.

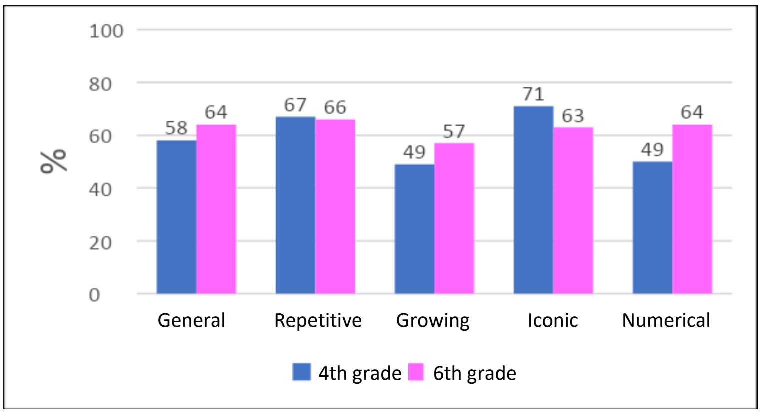
Within this analysis between the items, two of them caught our attention, namely items P1B and P5B, given the sharp drop in the percentage of students who got it right between items A and B, especially P1, whose percentage of correct answers fell from 87% (item A) to 38% (item B). This result leads us to conjecture that, although the majority of 6th graders identify the pattern of the sequence, they are not yet able to generalize the pattern identified.

In the next section, our final analysis will look at the comparison between the behavior of the two groups.

4.2 Inter-group performance

In this section we will present a comparative analysis of the performance of 4th and 6th grade students, considering (a) the types of sequences (repetitive and increasing), (b) the contexts (iconic and numerical) and (c) items A and B. The quantitative analysis of the data is based on the percentage of correct answers for each group studied, which are shown in Graph 1 for a clearer visual and comparative understanding of the data.

Graph 1: Overall average performance per class



Source: Research data (2023)

Based on the data shown in Graph 1, we can see that while the 4th grade students are competent in dealing with repetitive pattern sequences (67%) and iconic contexts (71%), the 6th graders show more homogeneous behavior, with a variation in the percentage of correct answers of less than 10%, but it is noteworthy that there is little difference between the overall percentage of correct answers from the 4th to the 6th grade. When we applied the Student's *t-student* ($t_{(83)}$ = -0,947, p = 0,346), we confirmed that there was no significant difference between the overall performance of these two groups.

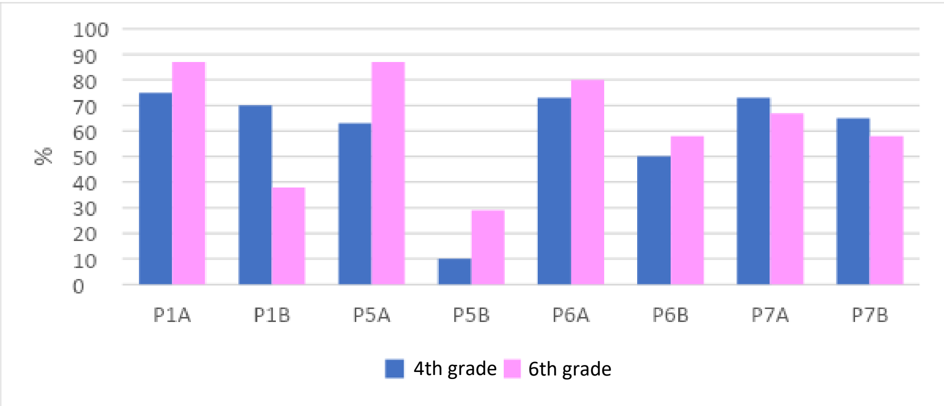
This result surprised us because we are talking about groups with different ages and levels of schooling. It was expected that the 6th grade group would actually do better than the 4th grade group, whether due to the school factor or the cognitive development factor.

We compared the performance of 4th and 6th graders using the *Chi-square test* for: (a) sequences of repetitive patterns, (b) sequences of increasing patterns and (c) sequences in an iconic context. In none of these cases was the difference between the results statistically significant. This allows us to conjecture that the behaviors of the two groups on the three points raised above are equivalent, with a high chance that the (few) percentage differences between them occurred by chance.

The only consistent difference between the behavior of the two groups was in their performance on number sequences. In this case, 6th graders showed a consistent difference of 15% more correct answers than 4th graders (X^2 = 7.34 p = 0.001). This result corroborates the findings of Porto (2018), who concluded that the numerical context is more favorable to 6th grade students. This can be attributed to the encouragement offered by the school for these students to use and understand numbers, while 4th graders are more encouraged to use visual representations and drawings.

Within this context, in order to gain a deeper understanding of these results, we conducted a comparative analysis of performance, based on an analysis of the problems (P1, P5, P6 and P7) and the hit rates on items A and B. However, where appropriate, we will present extracts from the students' protocols. This data is shown in Graph 2, allowing for a more detailed investigation of the differences observed between the 4th and 6th grade groups.

Graph 2: Performance of 4th and 6th graders on problems and items



Source: Research data (2023)

When we analyze the data shown in Graph 2, we see that in item A, both classes achieved an overall average of over 60%, which means that 2/3 of the classes got the next term requested in the question right. However, when we look at the performance of the two classes in relation to item B, we see that the 4th grade had an average performance of more than 50%, with the exception of P5. Year 6, on the other hand, performed above average on two problems (P6B and P7B), but showed a very large drop in performance on P1B and P5B.

In short, we can see that both had a low level of performance on P5. The 6th grade, in contrast to the 4th grade, showed a very large drop in relation to P1B. In order to understand the low performance of the two classes, we decided to introduce some discussions about the strategies adopted by the students in solving these problems, which we will do in the following section.

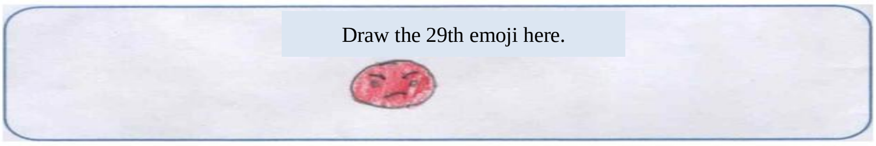
4.3 Analyzing some behaviors that can explain performance

We'll start by presenting some of the strategies presented by the 6th graders in P1B, as shown in Figure 1.

1. Look at the sequence of emojis below

1st 2s 3st 4st 5st 6st 7st 8st 9st 10st position

b. If I were to continue drawing the emojis in this sequence, which emoji would be in position 29?



Is there any way of knowing which emoji is in any position in this sequence? How? Answer:

Sim, só você ver qual carinha vem depois de carinha, porque está repetindo

Yes, you just have to see which emoji comes after the emoji, because it's repeating.

Extract from the student's protocol.

Figure 1: Extracts from the protocols of students 603 and 618 in P1 (Research data), 2023)

When analyzing the extract from student 603's[6] protocol, we noticed that the answer provided was *an angry face*, which would not be the correct answer. The angry face would be the element immediately after the laughing face (the last element drawn in the sequence). One possible hypothesis for this student's answer is that he didn't pay attention to the position requested (position 29), which is somewhat distant from the last position drawn in the sequence (9th), and simply drew the face in the next position (11th). Another possibility is that student 603 used the counting strategy, considering the ten positions presented in the sequence as a whole. Therefore, the 11th position would be the 1st position in the sequence and the 21st position would again be its 1st position. Our interpretation seems to be supported by this student's answer to item c: *"yes, you just have to see which face comes after the little face, because it's repeating itself"*. It's clear that he identifies a repeating sequence, but for him, the sequence consists of the ten elements presented that repeat and not three elements (happy face, angry face and sad face). In addition, the student doesn't seem to have understood that if the 3rd element of the sequence (the sad face) is in position 3, then all multiples of 3 will be *sad faces*. Position 21 is a multiple of 3, so its element is the *sad face*. Thus, we conjecture that the student used the counting strategy, considering the sequence with ten positions, without realizing that, in this case, position 10 and 11 would have the same face.

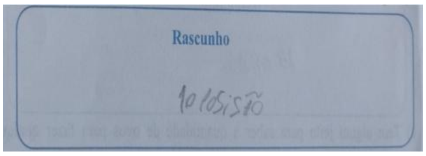
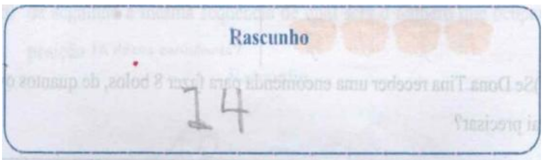
Next, we'll present examples of solutions to Problem 5 (P5), especially P5B. We chose this problem because, unlike P1, it was the one in which the students in both groups showed the lowest results in item B. It is a sequence that progresses from 1 to 1, but the element in the 1st position already starts with the number four. The students' protocols show that they had a hard time making the generalization. According to Lins and Gimenez (1997), this is a process that

[6] The first number indicates the student's school year, while the last two correspond to the order assigned by us.

is anchored in advanced reasoning, as it is not simply establishing instantaneous connections; deducing numerical patterns requires the student to establish processes that are not based on algorithms.

It seemed to us that this type of reasoning represents a challenge for elementary school students. This is reflected in more than 28% of blank answers or answers with no relation to the proposed problem. To better understand this performance, we will look at some of the strategies adopted by students from both groups in this P5. We'll start with the answers of students 638 (wrong) and 419 (wrong), shown in Figure 2.

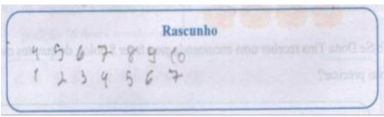
Figure 2: Extracts from the protocols of students 638 and 419, in P5B

<table>
<tr><td colspan="2">5. Look at the sequence of numbers below:
4 5 6 ...
1st 2 st 3 st 4 st position</td></tr>
<tr><td>Student protocol 6th grade — 638</td><td>Student protocol 4th grade — 419</td></tr>
<tr><td>Still following the same sequence, which number occupies the 10th position in this sequence?</td><td>Still following the same sequence, which number occupies the 10th position in this sequence?</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Source: Research data (2023)

When we looked at the students' protocols in Figure 2, we realized that both classes had difficulties solving this item. The 6th grade student didn't understand the question and ended up giving his own position as the answer. The 4th grade student, on the other hand, gave an answer of 14. We hypothesize that he used additive reasoning when adding up the position requested in item B and the last position represented in the problem (10 + 4 = 14). In this context, we realize that the failure to perform this item may be linked to a number of factors, including the wording of the question itself, such as the sequence starting with the number 4 (position 1), which may have led students to confuse the position and the element, despite it being clearly specified. In addition, the lack of understanding can be seen in the students' tendency to use the position or data presented in the problem as an answer, around 22% of the students used this strategy, this factor can be seen in the strategies of students 638 and 419, in Figure 3 below.

Figure 3: Extracts from the protocols of students 638 and 419, in P5B

<table>
<tr><td colspan="2">5. Look at the sequence of numbers below:
4 5 6 ...
1st 2 st 3 st 4 st position</td></tr>
<tr><td>Still following the same sequence, which number occupies the 10th position in this sequence?


Extract from Protocol 638</td><td>Still following the same sequence, which number occupies the 10th position in this sequence?


Extract from Protocol 419</td></tr>
</table>

Source: Research data (2023)

Of the students who got item P5B wrong, most used some element of the problem as a strategy. Student 638's excerpt shows that he started from the sequence given in the statement and followed it up to position 7, when he found that the element in position 7 was the numeral ten. Our hypothesis for this solution is that this student swapped the search for the element in position 10 for the element 10 found in position 7. Student 419's protocol, on the other hand, indicates that he created his own sequence - a repetitive pattern sequence - built from the three elements present in positions 1, 2 and 3. We can't say that student 419 is wrong in his procedure, since nowhere in the P5 statement does it say that this is an increasing pattern sequence.

Returning to the results presented by the 4th and 6th graders in our diagnosis, it is important to point out that, initially, we had hypothesized that the 6th graders would actually outperform the 4th graders. This is because: they are, on average, two years older than the 4th grade group, with cognitive development to their advantage; they have two more years of schooling, and are already in the final years of elementary school, studying math with specialist math teachers.

In addition, it is intriguing to note that both classes, according to the BNCC guidelines (Brasil, 2017), should have already been taught patterns in sequence, whether growing or repetitive, within the Algebra unit. With regard to patterns in sequence, the document establishes guidelines for students to be introduced to identifying and understanding these patterns from the earliest years. The teaching progression proposed by the BNCC allows students to have been exposed to and worked with different types of sequences by the time they reach 6th grade, preparing them for more complex levels of understanding and application of these patterns throughout their educational journey.

Given this context, we found in Jerônimo (2019) a question that we have adapted for our study, a question that deserves to be studied in the future: why do 4th grade students show similar results to 6th grade students, when the latter have been dealing with algebraic situations for at least 2 school years? A possible answer to this question may lie in the way 6th graders are introduced to algebraic situations, often through mechanical and decontextualized explorations. In addition, the author argues that this type of approach leads students to solve these situations in a disconnected way, using intuitive responses and, most of the time, without being able to interpret the proposed situations.

For Vale *et al.* (2011), it is essential that students start learning algebra in an intuitive and motivating way, which can be facilitated by real-world patterns. For the author, an important way of introducing it to students is through pattern analysis. This teaching method, based on identifying patterns in everyday life, provides a solid foundation for understanding algebra. By connecting abstract concepts with real-world situations, students not only develop a deeper understanding, but also cultivate a greater interest in mathematics.

5 Final considerations

The research aimed to analyze and compare the performance of 4th and 6th grade students in solving tasks involving Sequences of Repetitive and Increasing Patterns. The overall score revealed a difference between the average scores of the two groups, in favor of the 6th graders. However, this result was not statistically significant, which allows us to assume that it may have occurred by chance.

This finding suggests that, despite the differences in the average number of correct answers, the variation observed was still very small, if we consider the development between the two classes. This finding challenges our initial expectations, which assumed that the 6th graders would perform considerably better due to their educational and cognitive progress. These results may raise questions about the teaching methods used to understand the introduction of algebra in elementary school.

Likewise, when we compared the performances of the two groups in specific variables, such as sequence in the iconic context and increasing sequence, we identified a difference in favor of the 4th grade in the iconic context and in favor of the 6th grade in the increasing context, but in none of these comparisons was it statistically significant, so we can only comment on some trend in behavior. However, in the numerical sequence, there was a significant difference in favor of 6th graders, which we infer is due to 6th graders' greater familiarity with numbers than 4th graders.

When investigating the strategies used by 4th and 6th graders to solve problems, we identified the use of various strategies to solve problems, such as mental calculation, counting, using icons and recognizing regularities. Mental calculation was the most prevalent strategy in both years, followed by the use of icons. However, mental calculation proved to be a less effective strategy, especially for 4th graders, who made more errors. The use of icons also resulted in many errors, especially in Year 6, where students often created incorrect sequences of their own. In short, the effectiveness of the strategies varied depending on the problem and the school year, with one strategy being effective for one group but not the other.

In this way, we understand that the introduction of algebra as a thematic axis in the early years of elementary school, as provided for in the BNCC, is very recent. Therefore, we believe that this topic is very new in Brazil (it appeared in 2017) and is still unfamiliar to teachers, especially those in the early years. However, it represents a significant step forward in terms of global trends, as it has been addressed for decades by countries such as the USA, Spain and Portugal, among others. However, by incorporating algebra into the Brazilian curriculum from the first years of primary school, new problems arise, such as the need to prepare primary school teachers for this teaching.

In this sense, we have identified several gaps that could form the basis of future research. Firstly, there is a need to expand detailed studies on the teaching methods and pedagogical strategies used in the initial teaching of algebra, especially with students who have had no formal contact with this content. In addition, it is essential to carry out longitudinal research to follow the development of students over time, such as from 1st to 5th grade. Another important gap is the increase in research into teacher training, with a view to improving methodologies and strategies to help teachers teach algebra from new perspectives.

Therefore, a deeper analysis of the pedagogical methods used to teach algebra is necessary. This is especially important given the importance of continuity and progression in learning, according to the BNCC. This study can serve as a basis for future research that studies not only student performance, but also the teaching-learning processes of algebra in the early years, in order to find more effective strategies that meet educational needs.

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